Neighbor discovery time in schedule-based asynchronous duty cycling

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***Abstract*—In schedule-based asynchronous duty cycling, nodes alternate between active and inactive time slots in cycles that guarantee the occurrence of overlapping active time, therefore ensuring that neighbors will have opportunities to communicate. Among these schedules, Block Designs provide the minimal duty cycle for a given number of time slots [1]. However, there exists no precise estimation model for the resulting *NDT* when such schedules are employed. This paper provides an accurate model for nodes operating under Block Design schedules.**

I. Introduction

Duty cycling the radio is a fundamental energy-saving technique for mobile wireless devices, since the radio is often the most power hungry component in a node [2]. However, when duty cycling is applied to communicating devices, a coordination mechanism is needed to guarantee that nodes will find active neighbors during their own active time. While this can be achieved by synchronizing the nodes, with the aid of protocols or specialized hardware, coordination can also be attained asynchronously.

Said asynchronous duty cycling mechanisms are less costly in terms of computing and communication overhead. Within the category of asynchronous duty cycling, there are proposals that demand no traffic exchange and yet guarantee common active time. These schemes rely on the clever design of a wakeup schedule, which is repeated in cycles. If all nodes employ such schedule, there will be overlapping time slots, irrespective of their relative time offset.

In [1], the author proved that optimal asynchronous schedules (the one that provides the lowest duty cycle) for a given cycle length are achievable with *Block Designs* [3]. Since then, Block Designs have been proposed for duty cycling [4], [5], [6]. However, to the best of our knowledge, there is no accurate model for the Neighbor Discovery Time (*NDT*) when Block Designs are used to assign wakeup schedules.

A drawback of schedule-based asynchronous duty cycling is the increased neighbor discovery time that leads to increased communication latency, as nodes along a multi-hop path have to wait until the next node becomes active. Therefore, fully understanding the *NDT* of such schedules is of great importance. This paper provides a model for the estimation of the *NDT* for nodes operating under duty cycling schedules based on Block Designs.

Though there are models to estimate the *NDT*, they rely on two drastic simplifications: (1) they assume that the probability of message reception is 1 (perfect channel), which means that overlapping active time will always lead to neighbor discovery; and (2) they consider that the expected *NDT* equals the cycle length. We show that, by dropping these two assumptions, our model is more useful and realistic. The proposed model was validated with statistical simulations that indicate accuracy of 99% or better for most scenarios.

The rest of this paper is organized as follows. Section II introduces Block Designs. Section III presents our model for the *NDT*. Section IV shows the results of the validation tests. Section V discusses the results presented by the model while Section VI concludes the paper.

II. Block Designs

Symmetric Block Designs are combinatorial schemes that present interesting characteristics. A simple definition follows[1]:

Definition — Given a finite set V of v elements and integers $k, \lambda \geq 1$, a *Symmetric Block Design*, represented as $\{v, k, \lambda\}$, will have exactly v *blocks* ($B_0 \dots B_{v-1} \subset V$) of k elements and the following properties:

- Each and every element of V occurs in exactly k blocks;
- Any two blocks will have exactly λ elements in common.

As it generally happens in networking literature, we will refer to Symmetric Block Designs simply as Block Designs. For asynchronous duty cycling purposes, blocks correspond to the nodes' cyclic schedules, each element of the block being a time slot. From above, it is clear that nodes will have a fixed number (λ) of common active slots in each cycle. This property ensures that nodes will present common active slots irrespective of their time offsets or the blocks they operate on (as long as the blocks belong to the same Block Design). Moreover, [4] proves that two nodes operating under a $\{v, k, \lambda\}$ design will have overlapping active time equivalent to λ time slots, even if their slot borders are not aligned. Figure 1 shows a Block Design {7,3,1}.

III. NDT Model for Block Designs

Let A and B be two nodes operating under a scheme of asynchronous duty cycling based on a Block Design $\{v, k, \lambda\}$. As a simplification, suppose that A and B operate under different offsets (i.e. under different blocks). Define $e_i, 1 \leq i \leq \lambda$, as the *i-th* common active slot between both nodes within a given cycle (i.e. the *i-th* opportunity of discovery in a cycle). Figure 2 exemplifies such definitions with an instance

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[1]For a rigorous definition the reader is referred to [3]

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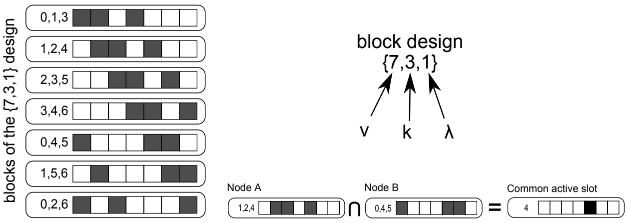

Fig. 1: Block Design {7,3,1} and its elements (blocks). Each of the seven (*v*) blocks will have three (*k*) elements and any two blocks will have exactly one (λ) common element. In this example, blocks [1,2,4] and [0,4,5] have element 4 in common.

TABLE I: Examples of Block Designs (selected from [7]).

$\{v, k, \lambda\}$	duty cycle
{7,3,1}	42.86%
{57,8,1}	14.04%
{183,14,1}	7.65%
{273,17,1}	6.23%
{1057,33,1}	3.12%
{4557,68,1}	1.49%
{9507,98,1}	1.03%

$\{v, k, \lambda\}$	duty cycle
{1023,511,255}	49.95%
{15,7,3}	46.67%
{11,5,2}	45.45%
{101,25,6}	24.75%
{400,57,8}	14.25%
{820,91,10}	11.10%
{4369,273,17}	6.25%

where two nodes operate under a {15,7,3} design. Clearly, for $0 < i < \lambda$:

$$E[e_{i+1} - e_i] = \frac{v+1}{\lambda+1} \quad \text{and,} \quad E[e_1] = \frac{v+1}{\lambda+1} - 1$$

In this case, we can calculate $E[NDT]$ from the definition of expectancy:

$$E[NDT] = \sum_{c=1}^{\infty} \sum_{i=1}^{\lambda} t_i^c \cdot p_i^c \tag{1}$$

where t_i^c is the time when the i_{th} discovery opportunity happens within cycle c and p_i^c is the probability that the discovery happens on that moment. But,

$$t_i^c = t_1^c + \sum_{j=1}^{i-1} \{e_{i+1} - e_i\}$$

$$E[t_i^c] = \left[v(c-1) + \left(\frac{v+1}{\lambda+1} - 1 \right) \right] + (i-1)\frac{v+1}{\lambda+1} \tag{2}$$

and,

$$p_i^c = p(1-p)^{\lambda(c-1)+i-1} \tag{3}$$

By substituting (2) and (3) in (1) and solving the summation, we find our model for the estimation of the *NDT* in asynchronous duty cycling based on Block Designs:

$$E[NDT] = \frac{v+1}{p(\lambda+1)} - \frac{(v+1)(1-p)^{\lambda} - (\lambda+1)}{(\lambda+1)[(1-p)^{\lambda} - 1]} \tag{4}$$

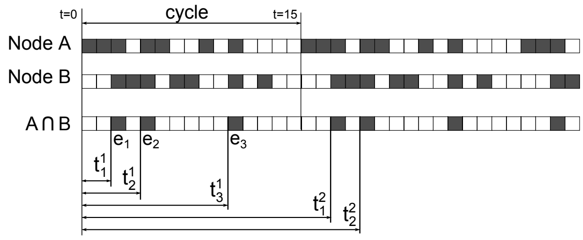

Fig. 2: Two nodes operating under a {15,7,3} design, with an offset of two slots.

IV. Model validation

To validate the proposed model, duty cycling schedules based on Block Designs were simulated in the R environment [8], which allowed for a great number of repetitions and the exact fixation of p (almost impossible to achieve in real scenarios) over a significant range of values — from 5% to 100 %, with increments of 5%. For each value of p and for *all known* Block Designs, 40000 measures were taken.

The results confirmed the accuracy of the proposed model. Figure 3 shows the results for the worst case found (the design {7,3,1}) and two other typical results (for designs {9507,98,1} and {4369,273,17}). Even for the worst case, the accuracy of the model is better than 99% for all cases where $p > 0.4$. Moreover, for most designs, the same accuracy is achieved also for lower values of p.

V. Discussion

To date, networking literature has paid attention mainly, if not exclusively, to a particular category of Block Designs where $\lambda = 1$, also known as *projective planes*. Projective planes provide optimal schedules in terms of duty cycle [1], but only one opportunity of discovery per cycle. Therefore, one interesting question is whether Block Designs with higher values of λ may or may not be advantageous, and under which circumstances.

As our model takes λ as a parameter, this comparison is made possible. The conclusion is that, while projective plans provide minimal duty cycle, the augmented frequency of opportunities caused by an increase in λ may reduce the *NDT* as the link quality deteriorates (low values of p). The comparison in Figure 4 illustrates that: the *NDT* for {57,8,1} (duty cycle 14.04%) is only shorter than the *NDT* of {400,57,8} (duty cycle 14.25%) while $p > 0.44$. Similar results are obtained from comparisons of other pairs, such as {820,91,1} vs. {91,10.1}, or {4369,273,17} vs. {273,17,1}. The curves for the *NDT* are remarkably close, and the projective planes perform only slightly better than other Block Designs in good quality links. In summary, Given the low availability of projective planes (35, in a total of 135 Block Designs in [7]), *non*-projective planes may be of use.

Some special cases can be obtained from Equation 4 and also help in the analysis of $E[NDT]$. Three of these cases are presented in Table II. Case 1 presents the behavior of $E[NDT]$ when $\lambda = 1$, i.e. for projective plans. In this case,

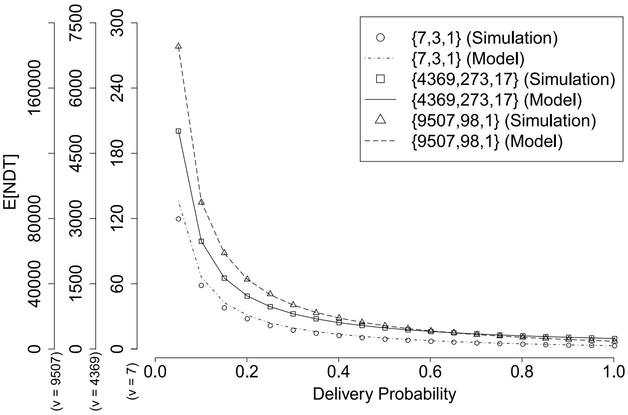

Fig. 3: Results of statistical simulations (points) in comparison to the proposed model (lines). Minor variations are noticeable only for small values of both p and v.

TABLE II: Four special cases obtained from the model

Case 1: $\lambda = 1$	Case 2: $\lambda = 1$ and $p = 1$	Case 3: $p = 1$
$E[NDT] = \frac{v}{p} - \frac{v+1}{2}$	$E[NDT] = \frac{v-1}{2}$	$E[NDT] = \frac{v-\lambda}{\lambda+1}$

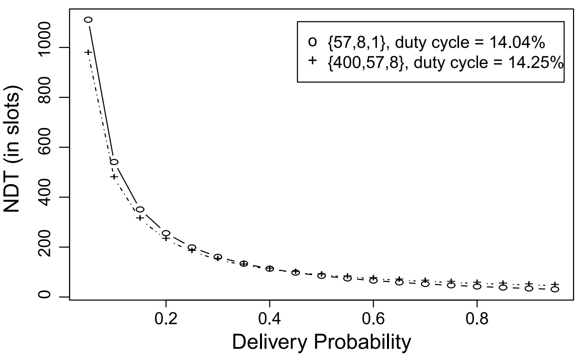

Fig. 4: A comparison between the NDT of $\{400,57,8\}$ and $\{57,9,1\}$ (both with similar duty cycles), shows that the better *NDT* depends on the link quality p.

as p increases, $E[NDT]$ tends to a little less than half a cycle ($\frac{v-1}{2}$). Case 2 is a particular case of Case 1 and presents an intuitive result for near-perfect links ($p \sim 1$): the waiting time will range from 0 (immediate) to v slots (a complete cycle) with mean equal to $v/2$. This comes from the fact that for $\lambda = 1$ and $p = 1$, *NDT* follows a discrete uniform distribution. Case 3 also considers perfect links (and can be extrapolated to near-perfect links without harm), but now there are many opportunities per cycle. The model shows that for a given cycle duration, designs with higher λ will reduce $E[NDT]$ at the expense of a higher duty cycle.

Finally, Figure 5 shows the extent to which our model differs from the common simplification in the literature, [5], [9] which assumes the *NDT* as equal to the cycle length, without taking p into account. The figure shows values for the $\{9507,98,1\}$, and it is qualitatively identical for all Block Designs. For values of $p < 0.66$, $E[NDT]$ is significantly higher than the cycle length.

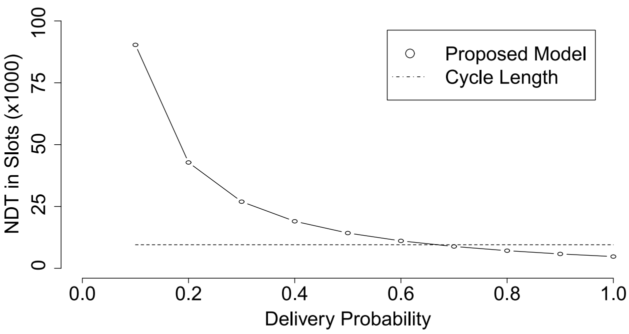

Fig. 5: *NDT* for the $\{9507,98,1\}$ Block Design, assuming our model or a fixed value equal to the cycle length.

VI. Conclusions

We presented a model for estimate of the Neighbor Discovery Time to be used in wireless networks operating under asynchronous duty cycling based on Block Designs. The model takes the parameters of the Block Designs as well as the probability of message reception as input and returns the expectancy for the *NDT*. Through statistical simulations, we demonstrated the high accuracy of the model (typically better than 99%).

We introduced Block Designs with large values of λ and established that they may be useful under certain circumstances, yielding slightly better results over marginal links. Finally, we also showed that, without a model such as the one presented, the latency resulting from asynchronous duty cycling would be overestimated (for high quality links) or grossly underestimated (for low quality links).

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