Probabilistic Spectrum Assignment for QoS-constrained Cognitive Radios with Parallel Transmission Capability

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Abstract—In this paper, we propose a multi-channel parallel transmission mechanism for channel assignment in cognitive radio (CR) networks. The proposed mechanism enables secondary users (SUs) to effectively utilize the spectrum while limiting the disruption rate at both SUs and primary users (PUs). The main novelty in our mechanism lies in considering the randomness of link-quality conditions and lifetime durations of idle channels to provide a statistical performance guarantee for SUs. This consideration results in improved spectrum utilization. Specifically, our mechanism attempts at minimizing the required spectrum resources for SU transmissions subject to minimum rate demand and minimum success probability requirements. Simulation results indicate that our proposed assignment satisfies the performance requirements at SUs. Results also show that utilizing the parallel transmission capability of CRs while considering the randomly time-varying nature of their operating environment allows for higher spectrum utilization and more energy saving.

I. INTRODUCTION

Recently, CRs are recognized as the key enabling technology to enhance spectrum utilization through opportunistic access to the licensed spectrum. Due to the randomness of PU channel availabilities and link-quality conditions, the operating environment of CR networks (CRNs) is characterized by their random time-varying nature. This dynamic and random nature poses many challenges related to development of distributed channel assignment/access mechanisms that capable of providing efficient SU communications [1]-[6]. Specifically, a CRN is expected to operate over a set of highly-separated frequency channels with different propagation characteristics and timevarying link-quality conditions. Worse yet, the availability of these channels and their lifetimes are also dynamically changing due to the randomness of PU activities [2]. Even after identifying an idle channel (or channels) and using it for a SU transmission, the SU may be required to immediately interrupt its transmission, vacate the operating channel (channels), and renegotiate the operating channels if new PUs become active. Therefore, the transmission time needed for a successful SU transmission can be quite critical. Thus, to improve spectrum utilization, SUs should select their operating channels while jointly considering the required transmission times over the selected channels and the randomness in their lifetime durations. Such consideration can significantly improve the packet success probability, which consequently reduces the average number of retransmission attempts for a successful packet delivery. This preserves more channels for potential future SUs, which reduces the SU blocking probability and allows for higher spectrum utilization and more energy saving.

In this paper, we develop a statistical approach by which SUs can opportunistically communicate over multiple channels while probabilistically guaranteeing their performance. Our approach leverages the unique capabilities of CRs while considering the peculiar characteristics of their operating environment. Specifically, the contributions of this paper are as follows. Based on generic stochastic models of PUs' activity, we first derive a general expression for the probability of success for a SU packet transmission over multiple channels. We show that the derived expression is a function of the instantaneous link-quality conditions and the statistical distribution of the availability durations of PU channels. Based on the derived expression, we formulate the spectrum assignment problem as an optimization problem with the objective of minimizing the required spectrum resources for a given SU transmission while satisfying pre-specified probability of success and rate demand requirements. We show that this optimization problem constitutes a binary linear programming (BLP) problem, which is, in general, NP-hard. Since computing the optimal solution for such problem grows exponentially with the number of idle channels, we develop a *polynomial-time* approximate algorithm for the BLP based on a sequential fixing procedure that provides a near-optimal solution. Through simulations, we show that the approximate solution is within 5% of the optimal one. The results indicate that our algorithm statistically satisfies the performance requirements under different traffic loads. Our simulation results also show that our assignment rsults to a significant performance improvement in terms of throughput and energy consumption over reference channel assignment algorithms. The rest of the paper is organized as follows. In section II-A, the network model is presented. An analytical expression for the packet success probability is derived in section II-B. In section III, the our problem is formulated. Section IV describes the proposed solution. In section V, we present numerical and simulation results. Conclusions are presented in section VI.

II. PRELIMINARIES

A. System Model

We consider a licensed spectrum containing $|\mathcal{K}|$ nonoverlapping channels of the same bandwidth. The status of channel *i* is modeled as an independent 2-state model alternating between BUSY and IDLE states. The BUSY period $(T_B^{(i)})$ represents the time that some PUs are transmitting over channel *i*. In this case, channel *i* cannot be used for SU transmissions. The IDLE period $(T_I^{(i)})$ represents the time that

no PUs are transmitting over channel *i*, and thus this channel can opportunistically be used by SUs. We assume that $T_B^{(i)}$ and $T_{I}^{(i)}$ are generally distributed independent random variables with finite means denoted by $\overline{T}_B^{(i)}$ and $\overline{T}_I^{(i)}$, respectively. Let $F_{T_B^{(i)}}$ and $F_{T_I^{(i)}}$ respectively denote the cumulative density functions (CDFs) of $T_B^{(i)}$ and $T_I^{(i)}$. At any given time, the *i*th channel is available with probability $P_I^{(i)} = \overline{T}_I^{(i)} / (\overline{T}_I^{(i)} + \overline{T}_B^{(i)})$. We also consider a distributed network of SUs, where each SU is equipped with n_r transceivers that can be used simultaneously. The n_r transceivers can be dynamically tuned to any idle channel in \mathcal{K} . For channel $i, \forall i \in \mathcal{K}$, the transmission power that a SU can use is 0 if the channel is busy, or $P_{\max}^{(i)}$ if channel *i* is idle, where $P_{\max}^{(i)}$ is the FCC regulatory maximum transmission power over channel *i*. In addition, the total transmission power over all selected channels for a given SU transmission is limited to $P_{\rm max}$, where $P_{\rm max}$ is the maximum power supported by the CR's battery. We assume that each SU can sense the spectrum and estimate the instantaneous interference over each channel. Each SU requires a minimum transmission rate, i.e., the aggregate data rate over all assigned channels should be greater than a prespecified transmission rate requirement. The probability of successfully transmitting a SU packet, denoted by P_{suc} , is our main quality of service (QoS) metric of interest. Because of the random nature of the operating environment of SUs, here we consider a soft success probability requirement in the form $P_{suc} \geq \gamma$, where γ is a given parameter.

B. Probability of Success Analysis

We now derive an expression for P_{suc} as a function of network parameters for a SU data packet of size L with a given channel assignment $\Omega = \{m_1, m_2, ..., m_{|\Omega|}\}$, where m_i represents the *i*th channel in Ω . Before proceeding further, we need to determine the distribution of the residual time duration of the various PU channels. Let the random variable $T_r^{(i)}$ denote the residual time duration of channel *i*. The CDF of $T_r^{(i)}$ can be computed in terms of the CDF of IDLE duration as [7]:

$$F_{T_r^{(i)}}(t) = \frac{\int_0^t \left(1 - F_{T_I^{(i)}}(t)\right) dt}{\overline{T}_I^{(i)}}.$$
 (1)

Let $\mathcal{M} \in \mathcal{K}$ and $\mathcal{R} = \{R^{(i)}, \forall i \in \mathcal{M}\}$ respectively denote the set of all idle channels sensed by communicating SUs and the set of achieved transmission rates over all channels in \mathcal{M} . Given the channel assignment $\Omega \subseteq \mathcal{M}$ and the transmission rates over the channels in Ω , the required transmission time over the assigned channels $(t_x^{(\Omega)})$ can be computed as:

$$t_x^{(\Omega)} = \frac{L}{\sum_{i \in \Omega} R^{(i)}}.$$
 (2)

To proceed in our analysis, we note that a packet transmission is considered to be successful over the channels in $\boldsymbol{\Omega}$ if the residual idle durations of all channels in Ω are greater than the required transmission time $t_x^{(\Omega)}$. Given the above and noting that IDLE durations of PU channels are statistically independent, the packet success probability over the selected channels in Ω ($P_{suc}^{(\Omega)}$) can be computed as:

$$P_{suc}^{(\Omega)} = \Pr\left(\min\left\{T_r^{(i)}, \forall i \in \Omega\right\} \ge t_x^{(\Omega)}\right)$$

$$= \Pr(T_r^{(m_1)} \ge t_x^{(\Omega)}, T_r^{(m_2)} \ge t_x^{(\Omega)}, ..., T_r^{(m_{|\Omega|})} \ge t_x^{(\Omega)})$$

$$= \prod_{i \in \Omega} \Pr\left(T_r^{(i)} \ge t_x^{(\Omega)}\right) = \prod_{i \in \Omega} \left(1 - F_{T_r^{(i)}}\left(t_x^{(\Omega)}\right)\right).$$

(3)

Substituting (1) into (3), $P_{suc}^{(\Omega)}$ can be rewritten in terms of $T_{\tau}^{(i)}$ as follows:

$$P_{suc}^{(\Omega)} = \prod_{i \in \Omega} \left(1 - \frac{\int_0^{t_x^{(\Omega)}} \left(1 - F_{T_I^{(i)}}(\tau) \right) d\tau}{\overline{T}_I^{(i)}} \right).$$
(4)

To make our analysis tractable, we model the status of a PU channel *i*, $\forall i \in \mathcal{K}$ as a Markov renewal process (MRP) that alternates between IDLE and BUSY states. This model was previously used in (e.g., [8]-[12]). This model can capture the temporal characteristics of PU channel availabilities. According to this model, IDLE (BUSY) durations for the various channels are statistically independent exponentially distributed random variables. In addition, for a given channel *i*, the durations of successive IDLE and BUSY durations are independent of each other. Based on this model and using (1), we can show that $F_{T_r^{(i)}}(t) = F_{T_I^{(i)}}(t) = 1 - e^{\frac{-i}{T_I^{(i)}}}$. By

substituting $F_{T_{r}^{(i)}}(t)$ into (3), $P_{suc}^{(\Omega)}$ can be expressed as:

$$P_{suc}^{(\Omega)} = \prod_{i \in \Omega} \exp\left(\frac{-t_x^{(\Omega)}}{\overline{T_I^{(i)}}}\right) = \exp\left(-t_x^{(\Omega)} \sum_{i \in \Omega} \frac{1}{\overline{T_I^{(i)}}}\right).$$
 (5)

For $i \in \mathcal{K}$, let x_i be a binary variable that is defined as:

$$x_i = \begin{cases} 1, & \text{if channel } i \in \Omega \\ 0, & \text{otherwise.} \end{cases}$$
(6)

By introducing the binary variable x_i , $P_{suc}^{(\Omega)}$ can be rewritten in terms of x_i as:

$$P_{suc}^{(\Omega)} = \prod_{i \in \mathcal{M}} \Pr(T_I^{(i)} \ge t_x^{(\Omega)} x_i) = \exp\left(-t_x^{(\Omega)} \sum_{i \in \mathcal{M}} \frac{x_i}{\overline{T}_I^{(i)}}\right)$$
$$= \exp\left(-\frac{L}{\sum_{i=1}^M R^{(i)} x_i} \sum_{i=1}^M \frac{x_i}{\overline{T}_I^{(i)}}\right).$$
(7)

where $t_x^{(\Omega)} = \frac{L}{\sum_{i \in \mathcal{M}} R^{(i)} x_i}$. We note that the expression in (7) will be used in formulating our channel assignment problem in Section III.

III. PROBLEM FORMULATION AND DESIGN CONSTRAINTS

Our objective is to optimize spectrum-utilization efficiency by minimizing the total spectrum resource needed for successful SU transmissions. Specifically, our treatment is targeted at a distributed CRN that uses CSMA/CA-like policy for control communications to resolve channel contention between different SU pairs. CSMA/CA-based protocols ensure that only one transmission (a SU transmitter-receiver pair) can access the control channel at any given time. For this SU transmission, the transmitter and the receiver need to select the minimum number of channels to use while meeting the following constraints:

C1. Hardware constraint: Each SU is equipped with n_r transceivers that can be used simultaneously, i.e.,

$$\sum_{i \in \mathcal{M}} x_i \le n_r. \tag{8}$$

C2. Received SINR and aggregate data rate constraints: The received signal-to-interference-noise ratio (SINR) over a selected channel *i* must be greater than a pre-specified threshold μ^* . This constraint can be ensured by setting $x_i = 0$ for any channel *i* with SINR < μ^* . In addition, the aggregate rate over all selected channels must be greater than or equal to a given rate demand R_D , i.e.,

$$\sum_{i \in \mathcal{M}} R^{(i)} x_i \ge R_D.$$
(9)

C3. Total transmit power constraint: For a SU transmission, the total transmission power (P_{tot}) over all selected channels is restricted to P_{max} , i.e.,

$$\sum_{i \in \mathcal{M}} P_{\max}^{(i)} x_i \le P_{\max}.$$
 (10)

where $P_{\rm max}$ is the maximum power supported by the CR's battery.

C4. Probability of success constraint: The probability of success $P_{suc}^{(\Omega)}$ for a given SU transmission must be greater than a pre-specified value γ . This constraint can be expressed as:

$$P_{suc}^{(\Omega)} = \exp\left(-\frac{L}{\sum_{i \in \mathcal{M}} R^{(i)} x_i} \sum_{i \in \mathcal{M}} \frac{x_i}{\overline{T}_I^{(i)}}\right) \ge \gamma.$$
(11)

By taking the natural log of both sides of (11) and algebraically manipulating the result, this constraint can be linearized as:

$$\sum_{i \in \mathcal{M}} c^{(i)} x_i \le 0 \tag{12}$$

where $c^{(i)} = \left(\frac{\ln \gamma}{L} R^{(i)} + \frac{1}{\overline{T}_{I}^{(i)}}\right)$.

Recall that our objective is to compute the optimal assignment Ω that uses the least spectrum resource (i.e., minimum possible number of channels) for a given SU transmission such that the performance requirements are guaranteed. Formally, our problem can be formulated as:

$$\begin{array}{ll} \underset{x_i}{\text{minimize}} & |\Omega| = \sum_{i \in \mathcal{M}} x_i \\ \text{subject to} & \textbf{C1-C4.} \end{array} \tag{13}$$

If multiple solutions exist for this optimization problem, we seek the one with the maximum aggregate rate. This can be ensured by adding the term $\left(\frac{-1}{\sum_{i\in\mathcal{M}}R^{(i)}}\sum_{i\in\mathcal{M}}R^{(i)}x_i=-\sum_{i\in\mathcal{M}}r_ix_i\right)$ to the objective function, where $r_i=\frac{R^{(i)}}{\sum_{i\in\mathcal{M}}R^{(i)}}$. Note that the introduced term is always < 1. Hence, for any two feasible assignment Ω_1 with $|\Omega_1|$ channels and Ω_2 with $|\Omega_2| > |\Omega_1|$ channels, our formulation will selects Ω_1 over Ω_2 , irrespective of their

aggregate transmission rates. It is clear that the optimization problem in (13) with the new objective function (i.e., $\sum_{i \in \mathcal{M}} (1 - r_i) x_i$) constitutes a BLP problem.

IV. A NEAR-OPTIMAL APPROXIMATION

Because our problem is a BLP problem, which is, in general, NP-hard, we develop a polynomial-time approximate algorithm to solve our BLP based on a sequential fixing procedure that provides a near-optimal solution. We note here that the use of sequential fixing-based suboptimal algorithms in solving integer programming problems were previously proposed and evaluated in several studies (e.g., [13], [14]). The key idea of our algorithm is to iteratively determine the binary variables x_i 's by solving a sequence of relaxed LP (RLP) problems with one x_i is finalized to a binary value in each iteration. The details of our sequential fixing algorithm is described as follows:

• The algorithm first sets x_i 's for all idle channels with SINR $< \mu^*$ to 0 and relaxes all unfixed x_i 's into real numbers in [0, 1].

• The algorithm then solves the resulting RLP. If the RLP is infeasible, then our BLP has no feasible solution (i.e., no feasible assignment). Otherwise, among the real-valued x_i 's solution to the RLP, the algorithm sets the largest to 1.

• The algorithm then checks the constraints C1-C4 assuming that all unfixed x_i 's are set to zero. If these constraints are met, the selected channel is our feasible assignment.

• Otherwise, at iteration j, $j = 2, ..., |\mathcal{M}|$, the algorithm relaxes all unfixed x_i 's to real values in [0, 1]. Then, it checks the feasibility region of the new RLP at iteration j. If this region is empty, this means the last fixed variable in the j-1 iteration should be changed to 0 and the jth RLP should be updated.

• The algorithm then solves the resulting RLP, whose variables are all unfixed x_i 's, and sets the largest x_i to 1.

• Given all fixed x_i 's at iteration j, the algorithm checks the constraints **C1-C4** assuming that all unfixed x_i 's are set to zero. If these constraints are met, the set of selected channels up to the *j*th iteration is the feasible channel assignment.

• This process is repeated until a feasible assignment is computed, or a total of $n_r x_i$'s are fixed to 1 (or all x_i 's are fixed, i.e., $j = |\mathcal{M}|$) and no feasible assignment can be found.

We note that our algorithm can determine a feasible solution or no feasible solution in no more than $|\mathcal{M}|$ iterations. Hence, its time complexity is bounded by the complexity of the LP solver times $|\mathcal{M}|$, which is polynomial.

V. PERFORMANCE EVALUATION

We first consider a single SU transmission, and investigate the accuracy of our algorithm through MATLAB simulations. We consider a licensed spectrum containing 20 nonoverlapping channels, each with bandwidth 2.5 MHz. The status of a PU channel is determined according to the 2-state MRP model described in Section II-A. The \overline{T}_I average idle durations for the 20 channels are 21, 51, 3, 21, 14, 2, 51, 14, 1, 21, 21, 51, 3, 21, 11, 2, 51, 14, 1, 21 ms, respectively. We set the idle probability $P_I^{(i)} = P_I$ for all channels. The transmission power for each PU is 0.5 Watt.



(a) Normalized cost w.r.t. the optimal (b) Normalized cost w.r.t. the optimal ($\gamma=0.85)$ $(\gamma=0.9)$



Fig. 1. Algorithm verification for different values of γ .

The simulation results are presented for 1000 "optimization instances" that can produce feasible solutions. For each instant, the source-destination distance is randomly generated from the range [20, 150] meter. Each SU generates 4-KB data packets, and requires a minimum data rate R_D and a minimum P_{suc} of γ . We set the SINR threshold to $\mu^* = 1$ dB, $P_{\max} = 1$ Watt, $P_{\max}^{(i)} = 0.25$ Watt, $\forall i \in \mathcal{K}$, and the thermal noise to 10^{-21} Watt/Hz. To describe the fading channel between any two SUs, a Rayleigh fading channel model with path loss exponent n = 4 is considered.

Fig. 1(a) and (b) plot the normalized cost obtained using our algorithm relative to the optimal cost obtained through exhaustive search for 100 instances for $n_r = 4$, $P_I = 0.5$ and for different values of γ . In most cases, our solution is identical to the optimal solution. Hence, our algorithm achieves a nearoptimal solution (within 5% of the optimal). Fig. 1(c) and (d) plot P_{suc} as a function of P_I for different values of R_D and γ . These figures¹ show that the bound on P_{suc} is always satisfied. Fig. 2 investigates the performance of our near-optimal algorithm (NearOpt) as a function of the idle probability P_I for different values of R_D and $\gamma = 0.85$ and 0.9. Our algorithm is compared with two multi-channel assignment algorithms: The link-quality-aware (referred to as MaxRate [11]) and lifetimeaware (referred to as MaxIdle). MaxRate (MaxIdle) sorts idle channels in a descending order of their data rates (lifetime durations), then it picks the minimum number of channels from the top of the sorted list such that the rate demand





Fig. 2. Performance of a single SU link: Average number of channels per received packet.

 R_D is satisfied. If the number of selected channels exceeds n_r or all idle channels cannot support R_D , or P_{suc} over the selected channels (computed using (5)) does not satisfy the requirement γ , then no feasible channel assignment can be found. Fig. 2 reveals that NearOpt requires less number of channels for a successful packet transmission, which results in improved spectrum utilization. Fig. 3 plots the average energy consumption for a successful packet transmission for $\gamma = 0.85$ and 0.9. It is clear that the performance of our NearOpt significantly outperforms the performance of the other algorithms in terms of energy consumption.

We now evaluate the performance of our algorithm in a multi-user environment via simulations. Specifically, we simulate a CRN with N SUs distributed over a 150 x 150 meter² area. We set $n_r = 3$. To resolve channel contention between SUs, we adopt the multi-channel CSMA/CA MAC protocol presented in [11]. As shown in Figs. 4 and 5, NearOpt achieves higher network throughput and and lower energy consumption than the other two algorithms under different SU and PU traffic loads. Other results indicate that the bound on P_{suc} is always satisfied for all algorithms. In summary, our algorithm allows for better spectrum utilization and more energy saving.

VI. CONCLUSION

We proposed a multi-channel parallel transmission mechanism for channel assignment in CRNs. Our mechanism consid-

¹The performance of our algorithm is comparable to the one for the optimal solution. Hence, for clarity, the optimal solution is not shown in our figures.



Fig. 3. Performance of a single SU link: Average energy consumption.

ers the randomness of link qualities and lifetime durations of PU channels to provide a statistical performance guarantee for SUs. Simulation results showed that this consideration results in improved spectrum utilization and more energy saving.

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(a) $\gamma = 0.85$



Fig. 4. Throughput performance vs. number of SUs for different values of γ and $R_D = 25$ Mbps (similar behaviors were observed for different values of R_D).



Fig. 5. Energy Consumption vs. number of SUs for $\gamma = 0.9$ and $R_D = 25$ Mbps (similar behaviors were observed for different values of γ and R_D).

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