Calculating the Maximum Throughput in Multihop Ad Hoc Networks*

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Abstract. This paper analyzes the communication between two points in wireless ad hoc networks operating at the IEEE 802.11 standard mode. The path from the source to the destination consists of a chain of nodes. We aim at deriving an analytical expression for the maximum throughput. We show that the maximum throughput can be increased, under certain conditions, through the use of two paths: the shortest path and an appropriately chosen alternative path that takes into account the interference problem. The alternative path strategy allows a gain up to 50% of the maximum throughput achieved in the single path strategy.

1 Introduction

Nowadays, the IEEE 802.11 standard is widely deployed in local wireless networks. In the ad hoc mode, it implements a distributed medium access mechanism called DCF (Distributed Coordination Function), which applies the CSMA/CA (Carrier-Sense Multiple Access/Collision Avoidance) access method.

The hidden terminal problem is a classical challenge in wireless networks. The DCF proposes the use of RTS (Request To Send) and CTS (Clear To Send) frames to solve this problem. By means of an RTS frame, the sender shows all his neighbors the intention to transmit and the receiver allows the transmission by sending a CTS frame, showing that its neighborhood is free.

Xu *et al.* [1] showed that the RTS/CTS handshake cannot completely solve the hidden terminal problem, due to the effect of the interference. This mechanism assumes that all nodes that could interfere with the frame reception (which will be called hidden nodes) are able to receive the CTS too. Xu *et al.* [1] derived an expression showing that the interference range is a variable range depending on the distance from the sender to the receiver and the signal to interference ratio at the receiver. Moreover, they showed that when the distance from the sender to the receiver is longer than a threshold value, the interference range becomes greater than the transmission range. As a consequence, we cannot assume that the CTS frame is received by a hidden node.

Gupta *et al.* [2] and Li *et al.* [3] analyzed the capacity of ad hoc networks. They showed that the capacity depends on some local radio parameters, the MAC (Medium Access Control) protocol, the network size, and the traffic patterns. Saadawi *et al.* [4] considered the performance of the IEEE 802.11 MAC protocol in multihop ad hoc networks, through the analysis of a TCP (Transmission Control Protocol) traffic.

Li *et al.* [3] analyzed the capacity of a chain of nodes considering the interference range. They showed the maximum utilization achievable for a fixed interference range

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in a specific chain case. In a companion paper [5], we analytically generalized an expression for the maximum utilization of a chain, considering a variable interference range and also considering all the possible relevant distances between two neighbors in the chain. As it was assumed by Li *et al.* [3], we assumed [5] a simplification, considering the carrier sensing range equal to the transmission range. As a matter of fact, this value, which depends on the sender equipment, is typically greater than the transmission range. Accordingly, in this work, we allow the carrier sensing range to assume greater values and we realize that it may have an important impact on the maximum throughput. Although more complex, this model is more complete and precise. Again we generalize an expression for the maximum throughput, now taking into account both the carrier sensing and the interference effects.

We showed in [5] that the maximum utilization can be increased through the simultaneous use of two paths to the destination: the shortest path and an appropriate alternative path that takes into account the interference problem. In the present work, we also generalize an expression for the maximum utilization achievable using an alternative path, now considering both the carrier sensing and the interference effects.

In Section 2, we present the effects of the interference and carrier sensing. In Section 3, we analyze the maximum utilization of a chain of nodes. In Section 4 we analytically show that the simultaneous use of the shortest path and an alternative path can improve, under certain conditions, the throughput. Section 5 presents our conclusions.

2 The effects of the interference and the carrier sensing

In order to correctly receive a packet, the signal power at the receiver must be strong enough. Hence, the signal to noise plus interference ratio must be greater than the minimum value specified for the receiver equipment. Increasing the distance from the sender to the receiver results in the reduction of the signal power at the receiver, which means that nodes more distant from the receiver can become hidden nodes.

Let d be the distance from the sender to the receiver, r be the distance from the receiver to a third node that might want to transmit, and SIR_{TH} be the minimum value for the signal to interference ratio required for a successful reception. Hence Xu *et al.* [1] showed that every node separated by less than $d\sqrt[4]{SIR_{TH}}$ meters from the receiver can indeed interfere with its reception.

Let R_{Tx} be the transmission range. Xu *et al.* [1] showed that, when the sender and the receiver are more than $\frac{1}{\sqrt[4]{SIR_{TH}}}R_{Tx}$ meters away from each other, the RTS/CTS handshake does not solve the hidden terminal problem.

According to the CSMA/CA mechanism, every node, that wants to transmit some data, before sending it, should sense the medium. This procedure may help avoiding the hidden terminal problem. That happens because the carrier sensing range, whose value is fixed and specified by the sender equipment, is typically greater than the transmission range. Let R_{CS} be the carrier sensing range, Xu *et al.* [1] showed that the hidden terminal problem is completely solved [1] when $R_{CS} = (1 + \sqrt[4]{SIR_{TH}})R_{Tx}$. Increasing R_{CS} beyond this value is not necessary. Moreover, increasing R_{CS} also increases the number of nodes that could transmit without interfering with the initial transmission, but they are prevented from doing it, due to the carrier sensing. Therefore the R_{CS} should not be too high, because this way it reduces the network capacity.

3 The capacity of a chain of nodes

The throughput experienced by a source of traffic is maximized when the destination is in the transmission range of the source, situation called "direct communication", and the medium is free. As an example, Li *et al.* [3] showed that, even in this best case, the IEEE 802.11 at a 2 Mbps rate is only able to manage a maximum throughput of 1.7 Mbps for 1500 bytes of packet size. The data rate is also reduced for the IEEE 802.11 standard operating at others rates, like 11 Mbps or 54 Mbps. The reduction is due to the overhead added by the RTS/CTS/ACK exchange, to the preamble sent before the transmission of each MAC frame, to the interframe spaces, and to the *backoff* mechanism. Furthermore, the maximum throughput depends on the data packet size.

The scenario with only two neighboring nodes gives an upper bound for the maximum throughput, which can be taken as a baseline for comparison. So, in the following analysis, the maximum utilization of a general case is a fraction of this reference value [3,5]. The general case is a "multihop communication". In this case, the nodes have to cooperate, forwarding the packets from the source to the destination.

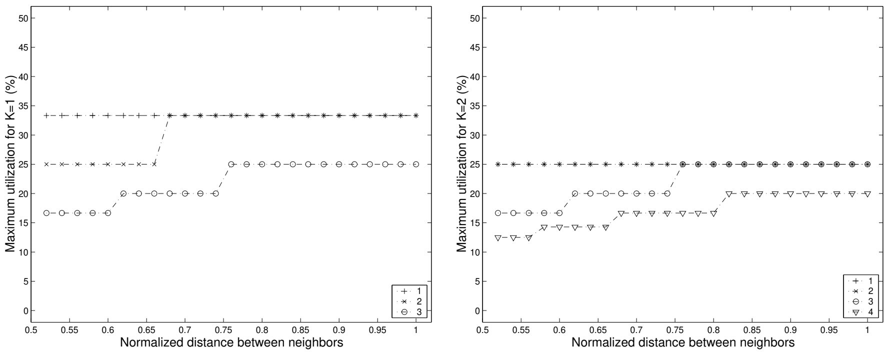
Let U_{max} be the maximum utilization. Now we will analyze the maximum utilization of a chain of nodes P_S, P_1, P_2, ..., P_D when the distance d between neighbors is such as $\frac{R_{Tx}}{2} < d \leqslant R_{Tx}$. So it can be written that $d = L \times R_{Tx}$, with $0.5 < L \leqslant 1$. This constraint arises because if $d > R_{Tx}$, then it is impossible reaching the destination and if $d \leqslant \frac{R_{Tx}}{2}$, then the nodes would not send the packets to their closer neighbors, instead they would send them to their neighbors closer to the destination, minimizing the number of hops. Both the carrier sensing and the interference caused by hidden nodes play an essential role on the analysis of the maximum utilization, which will be accomplished in three steps. In the first one, our analysis will consider only the carrier sensing effect, letting R_{CS} assume different values, and it will be called $U_{max_{CS}}$. In the second one, we will calculate $U_{max_{Int}}$, considering only the interference effect [5]. In this case, SIR_{TH} may assume different values. And in the last step, we will find a single value, U_{max}, based on the results of the previous steps. In this paper, the use of the term maximum utilization without any other specification implicitly means that the interference and the carrier sensing range were considered.

Let R_{CS} be the carrier sensing range. It is well known that $R_{CS} \geqslant R_{Tx}$. Hence, $R_{CS} = V \times R_{Tx}$, with $V \geqslant 1$. Let I be the number of nodes that can sense a transmission from the source. Hence, $I = \lfloor \frac{R_{CS}}{d} \rfloor = \lfloor \frac{V}{L} \rfloor$. If we assume that the source is transmitting, then none of its successors until P_I may transmit simultaneously, since the medium is sensed as busy by these nodes. But P_{I+1} (and its successors in the chain) does not sense the transmission from the source and it can send a frame concomitantly. Therefore $U_{max_{CS}} = \frac{1}{I+1}$.

Let R_{Int} be the interference range and let K be the number of nodes located in the interference range of the source. Then $K = \lfloor \frac{R_{Int}}{d} \rfloor = \lfloor \sqrt[4]{SIR_{TH}} \rfloor$. The reception by P_1 of a frame from P_S can be interfered by the transmission of all the successors of P_1 until the node P_{K+1}. Moreover, P_{K+2} and its successors in the chain cannot interfere with the transmission of P_S. Therefore $U_{max_{Int}} = \frac{1}{K+2}$. It only depends on K, which means it only depends on the SIR_{TH}, whose value must be greater than 1.

As both the carrier sensing and the interference effects play an important role on the definition of the maximum utilization, we must consider both of them together. So the maximum utilization assumes the minimum value between $U_{max_{CS}}$ and $U_{max_{Int}}$. This way, U_{max} is given by the dominant effect, depending on R_{CS}, d, and SIR_{TH}.

In the Figures 1(a) and 1(b), it is presented the U_{max} value when $1 < SIR_{TH} < 16$ and $V \in \{1; 2; 3\}$ and when $16 \leqslant SIR_{TH} < 81$ and $V \in \{1; 2; 3; 4\}$, respectively. The maximum value of V in each case was chosen, in order to illustrate a limit case, since Xu *et al.* [1] showed that when $R_{CS} = (1 + \sqrt[4]{SIR_{TH}})R_{Tx}$, the hidden terminal problem is completely solved and that a greater value of R_{CS} is useless.



(a) $V \in \{1; 2; 3\}$ and $K = 1$

(b) $V \in \{1; 2; 3; 4\}$ and $K = 2$

Fig. 1. Maximum utilization.

Once defined the SIR_{TH}, $U_{max_{Int}}$ remains constant for $\frac{R_{Tx}}{2} < d \leqslant R_{Tx}$ and it defines the upper bound of $\frac{1}{3}$ and $\frac{1}{4}$ in the Figures 1(a) and 1(b), respectively. The existence of these limits explains why some curves overlap partially, in the Figure 1(a) and in the Figure 1(b) when $U_{max_{CS}} \geqslant U_{max_{Int}}$. The $U_{max_{CS}}$ value depends on the ratio between R_{CS} and d. Increasing this ratio, the carrier sensing effect may become dominant, reducing U_{max} some more, as it can be seen in the Figures 1(a) and 1(b).

4 The use of an alternative path

As presented in the Figure 2, let us suppose there is a second path (P_S, A_1, A_2, …, P_D) to the destination, called alternative path. This path is longer, then a routing protocol, which uses only one path per destination implementing a minimum number of hops metric, will continue using the path P_S, P_1, P_2, …, P_D, called primary path. Aiming at increasing the throughput [5], the flow can be split in two parts, as it is shown in the Figure 2, using two different paths: the primary and the alternative path. The source injects frames into the network, alternating each frame into each path.

Once defined the carrier sensing range (R_{CS}), if P_S is transmitting, then all the nodes separated from P_S by a distance less than or equal to R_{CS} will be prevented from transmitting, because they sense the medium as busy. That holds true for nodes in the primary path and also for nodes in any other alternative path that may exist. If we define the maximum utilization using an alternative path and considering only the carrier sensing range as $Alt_{max_{CS}}$, then $Alt_{max_{CS}} = U_{max_{CS}} = \frac{1}{I+1}$.

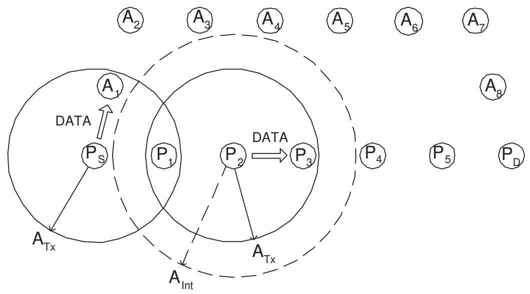


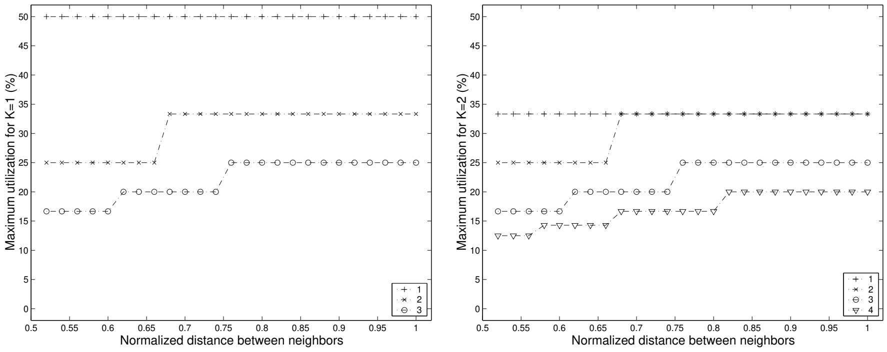
Fig. 2. A primary and an alternative path.

Nevertheless, we showed [5] that the maximum utilization, using an alternative path and considering only the interference effect, now defined as $Alt_{max_{Int}}$, can be increased when compared to $U_{max_{Int}}$. Again, let us assume that the neighboring nodes in each path are separated by a distance d, such as $\frac{R_{Tx}}{2} < d \leqslant R_{Tx}$. The function $dist(P_A, P_B)$ gives the distance from the node P_A to P_B. Again, $K = \lfloor \sqrt[4]{SIR_{TH}} \rfloor$, which is the number of nodes in the interference range of the source. In the chain case, P_S and P_{K+1} cannot transmit simultaneously, since $dist(P_1, P_{K+1}) \leqslant R_{Int}$. Aiming at obtaining a value for $Alt_{max_{Int}}$ greater than $U_{max_{Int}}$, now we allow this concomitant transmission, if P_S can send the frame to a neighbor A_1, such as $dist(A_1, P_{K+1}) > R_{Int}$. An equivalent condition should be respected by P_1 and A_{K+1}. An analogous procedure is recommended at the end of the process, when the two paths get closer to each other. And in the middle of the chain, we recommend that a distance greater than R_{Int} should be guaranteed. This procedure, called $Proc_{Alt}$, is exemplified in the Figure 2, which considers only the interference effect. In the Figure 2, we have $1 < SIR_{TH} < 16$. In this case, P_S can transmit to A_1 simultaneously with the transmission from P_2 to P_3.

We showed in [5] that using only one alternative path with $Proc_{Alt}$ is enough for achieving the optimum throughput. In a general case, $Proc_{Alt}$ allows the simultaneous transmission of P_S and P_{K+1} (and also of P_S and A_{K+1}). So $Alt_{max_{Int}} = \frac{1}{K+1}$ [5].

Let us define the maximum utilization using an alternative path and considering both the carrier sensing and the interference effects as Alt_{max}. Then the value of Alt_{max} is the minimum between $Alt_{max_{CS}}$ and $Alt_{max_{Int}}$. The Figures 3(a) and 3(b) present Alt_{max}, when $1 < SIR_{TH} < 16$ and $V \in \{1; 2; 3\}$ and when $16 \leqslant SIR_{TH} < 81$ and $V \in \{1; 2; 3; 4\}$, respectively. Once specified the SIR_{TH}, $Alt_{max_{Int}}$ defines the upper bound of $\frac{1}{2}$ and $\frac{1}{3}$ in the Figures 3(a) and 3(b), respectively.

The maximum utilization can be increased with the alternative path strategy when, in the chain case, we have $U_{max_{CS}} > U_{max_{Int}}$; otherwise, the carrier sensing defines the maximum utilization and the use of $Proc_{Alt}$ cannot change its value. Therefore we have $Alt_{max} > U_{max}$ when $\lfloor \frac{V}{L} \rfloor < K + 1$. The gain in percentage achieved by the alternative path strategy when compared to the chain case can be generalized as $100 \left(\frac{Alt_{max_{Int}}}{U_{max_{Int}}} - 1 \right) = 100 \left(\frac{K+2}{K+1} - 1 \right)$. When $K = 1$, it is possible achieving the maximum gain, which is 50%. Such gain is achievable for $V = 1$ and any value of d or for $1 \leqslant V < 2$ and values of d, such as $L > \frac{V}{2}$. When $K = 2$, the gain is 33% and it is achievable for $1 \leqslant V \leqslant 1.5$ and any value of d or for $1.5 < V < 3$ and $L > \frac{V}{3}$.



(a) $V \in \{1; 2; 3\}$ and $K = 1$

(b) $V \in \{1; 2; 3; 4\}$ and $K = 2$

Fig. 3. Maximum utilization implementing the alternative path.

5 Conclusions

Due to the characteristics of the IEEE 802.11 medium access mechanism, the communication between two nodes through a chain of nodes has serious access constraints. The maximum utilization of a chain is $\frac{1}{3}$ in the best case. This situation becomes even more restrictive if the signal to interference ratio necessary for a correct reception (SIR_{TH}) is greater than or equal to 16, due to the hidden terminal problem. The presence of the interference gives an upper bound for the maximum utilization. In addition, we must consider the carrier sensing effect. The ratio between the carrier sensing range and the distance d between neighboring nodes is a relevant parameter. Increasing this ratio, namely increasing R_{CS} or reducing d, the carrier sensing effect may become dominant in the definition of the maximum utilization of the chain, reducing it even more.

We show that it may be possible to increase the maximum throughput, using multiple paths. Therefore we simultaneously use the shortest path and an appropriately chosen alternative path, which takes into account the interference problem. Only one alternative path is enough for achieving the optimum throughput. When $1 < SIR_{TH} < 16$, the use of the alternative path allows a gain of 50% in the maximum utilization when compared to the chain case. When $16 \leqslant SIR_{TH} < 81$, the allowed gain is 33%.

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