Comments on "Evaluation of the Mean Error-Free Interval of a Noisy Data Channel"

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Abstract—**This paper corrects a few results reported in [1]. It shows that, for very long data strings, the Mean Error-Free Interval as a function of the packet error rate P is not discontinuous. In addition, the corrected results are extended to point-to-multipoint environment.**

In [1], Georganas *et al.* derived expressions for the Mean-Error Free Interval (MEFI) in a data string of B packets in a packet-oriented data transmission system. An Error-Free Interval (EFI) was defined as the number of consecutive error-free packets between two packets in error. Expressions were derived for the conditional (given that i packets out of B are in error) and unconditional MEFI's. However, as will be shown in this letter, the evaluation of these MEFI's must take into account that the length K of an EFI can be equal to zero and not only for $K = 1, 2, \ldots, B-i$. An EFI of length zero corresponds to the interval between two contiguous packets in error.

The notation and assumptions utilized in this correspondence are identical to those in the mentioned paper. Thus, the packets have L bits, including overhead, the Bit Error Rate (BER) is p, the errors are statistically independent, the probability of a packet being in error is:

$$P = 1 - (1-p)^L \tag{1}$$

and the probability of i packets being in error among the total B is given by the binomial distribution:

$$(PR)_i = \binom{B}{i} P^i (1-P)^{B-i}. \tag{2}$$

It was shown in *Proposition 1* of [1] that, for given B and i, the number of EFI's of length K, $K = 1, 2, \ldots,$ B$-i$, is given by:

$$N(K) = (i+1)\binom{B-K-1}{i-1} \;;\quad 1 \le K \le B-i. \tag{3}$$

Using the same steps utilized in [1], it is not difficult to show that equation 3 also is valid for $K = 0$. Hence, equation 3 becomes valid for $0 \le K \le B-i$.

This correction implies that the expressions for $\overline{E}(B,i)$ (equation 5), $\overline{E}_1(B)$ (equation 8) and $\overline{E}_1(\infty)$ (equations 9 and 10) in [1] must also be corrected.

The corrected expression for the Mean Error-Free Interval given that i packets out of B are in error ($\overline{E}(B,i)$ in *Proposition 2* of [1]) is:

$$\overline{E}(B,i) = \frac{B-i}{i+1} \tag{4}$$

Proof:

$$\overline{E}(B,i) = \frac{\sum_{K=0}^{B-i} K(i+1)\binom{B-K-1}{i-1}}{\sum_{K=0}^{B-i}(i+1)\binom{B-K-1}{i-1}} = \frac{\sum_{K=1}^{B-i} K\binom{B-K-1}{i-1}}{\sum_{K=0}^{B-i}\binom{B-K-1}{i-1}}. \tag{5}$$

As [2]

$$\sum_{K=0}^{B-i}\binom{B-K-1}{i-1} = \binom{B}{i}, \tag{6}$$

equation 5 becomes

$$\overline{E}(B,i) = \frac{\binom{B}{i+1}}{\binom{B}{i}} = \frac{B-i}{i+1}. \tag{7}$$

The corrected expression for the unconditional Mean Error-Free Interval in a data string of B packets, measured in packet lengths ($\overline{E}_1(B)$ in *Proposition 3* of [1]), is:

$$\overline{E}_1(B) = \frac{1-P}{P}[1-(1-P)^B]. \tag{8}$$

Proof:

$$\begin{aligned}
\overline{E}_1(B) &= \sum_{i=0}^{B}\overline{E}(B,i)(PR)_i \\
&= \sum_{i=0}^{B-1}\frac{B-i}{i+1}\binom{B}{i}P^i(1-P)^{B-i} \\
&= \frac{1-P}{P}\sum_{j=1}^{B}\binom{B}{j}P^j(1-P)^{B-j} \\
&= \frac{1-P}{P}[1-(1-P)^B].
\end{aligned}$$

Moreover, the corrected expression for $\overline{E}_1(\infty)$, the Mean Error-Free Interval, for a very long data string ($B \to \infty$) (*Corollary 1* of [1]) is:

$$\overline{E}_1(\infty) = \frac{1-P}{P};\quad 0 \le P \le 1. \tag{9}$$

The same result can be obtained by a different way: given that a packet has been received in error, the probability of the EFI length, between itself and the next packet in error, being $k = 0, 1, 2, \ldots$ is $Pr(k) = (1-P)^k P$. Therefore, the mean length of EFI is:

$$\overline{E}_1(\infty) = \sum_{k=1}^{\infty} kP(1-P)^k = P\frac{1-P}{P^2} = \frac{1-P}{P}. \tag{10}$$

Fig. 1 shows $\overline{E}_1(\infty)$ as function of P, for both the old and the corrected expressions. As can be observed the new expression does not present the discontinuity presented by the expression of $\overline{E}_1(\infty)$ in [1]. For $P \ll 1$ the difference between these expressions of $\overline{E}_1(\infty)$ is negligible.

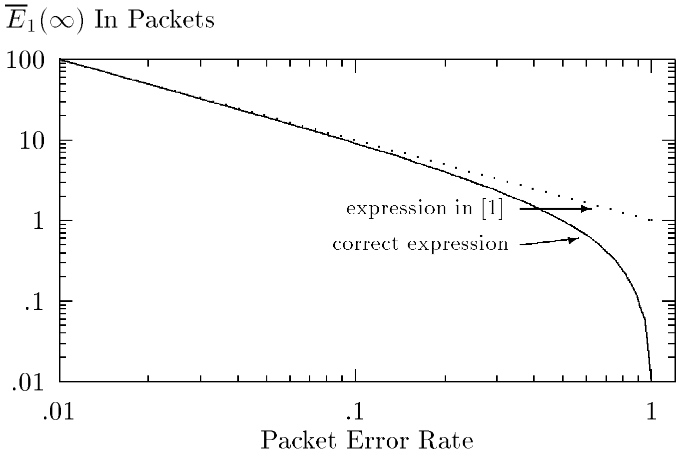

Fig. 1 - $\overline{E}_1(\infty)$ *as function of P*

Furthermore, the expression for this Mean Error-Free Interval, as a function of the Bit-Error Rate and of the packet length is:

$$\overline{E}_1(\infty) = \frac{(1-p)^L}{1-(1-p)^L}. \tag{11}$$

Fig. 2 shows original and corrected plots of $\overline{E}_1(\infty)$ as a function of L, for various p values. It can be observed that $\overline{E}_1(\infty)$ reaches values lesser than 1 (one) when L nears of $1/p$ and tends to 0 (zero) as L becomes greater than $1/p$.

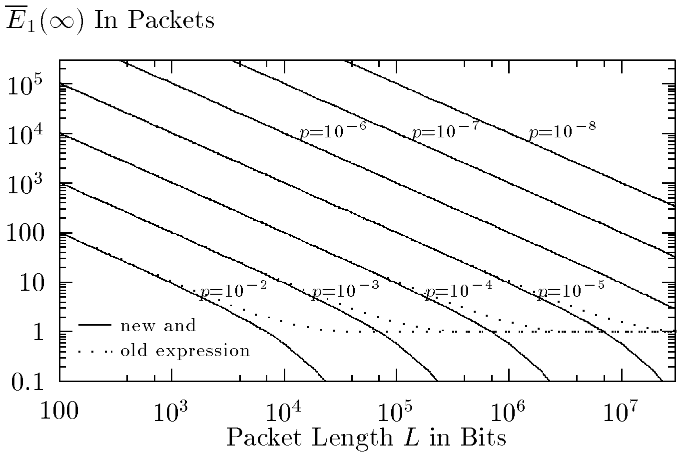

Fig. 2 - $\overline{E}_1(\infty)$ *as function of L*

Point-to-Multipoint environment

Finally, the above results are extended to point-to-multipoint channels. Considering an environment with R receivers, the probability of a packet being received in error by at least one receiver is:

$$P = 1-(1-p)^{L.R}. \tag{12}$$

Moreover, the probability of i packets being in error among the total B is also given by equation 2, but with P given by equation 12.

Clearly, equation 4 also is valid for multipoint channels, since that result does not depend on the probability of a packet being in error. At the same time, it is not difficult to show that equations 8, 9 and 10 are also valid for point-to-multipoint channels, with P given by equation 12. On the other hand, $\overline{E}_1(\infty)$ can be expressed as a function of the Bit Error Rate (p), the packet length (L) and of the number of receivers (R):

$$\overline{E}_1(\infty) = \frac{(1-p)^{L.R}}{1-(1-p)^{L.R}}. \tag{13}$$

This result shows that the plots of $\overline{E}_1(\infty)$ as a function of the product $L.R$, for various BER's (p), are equal to those in Fig. 2, only replacing L by the product $L.R$ in x axis. It means that the value of $\overline{E}_1(\infty)$ in a point-to-multipoint channel with 10 receivers, for example, is equal to that value in a point-to-point channel with packets 10 times greater.

References

[1] N. D. Georganas, S. Shiva, P. K. Verma, and J. S. Jawanda, "Evaluation of the mean error-free interval of a noisy data channel," *IEEE Transactions on Communications*, vol. COM-26, pp. 185–187, Jan. 1978.

[2] J. Riordan, *Combinatorial Identities.* John Wiley and Sons, 1968.