# Performance of Repeated Retransmission GB(N) Schemes for Point-to-Multipoint Noisy Channels

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ABSTRACT This paper presents and evaluates two point-to-multipoint GB(N) protocols. They accomplish continuously repeated retransmission to overcome noisy channel error condition and provide buffer at receivers, aiming at improving their throughput efficiencies. The first scheme is a point-to-multipoint version of the single destination Morris's GB(N). The other one results from slight modifications in the retransmission procedure. After the constinuously repeated retransmission, the transmitter sends $m$ ($m \geq 1$) copies of messages sent in advance and received in error. Numerical results show that the proposed modification significantly improves the throughput efficiency. Moreover, the proposed protocols outperforms many other multidestination GB(N) schemes.

## INTRODUCTION

Point-to-multipoint communication systems, in which a single transmission from a source station may be received simultaneously by several destination ones, make economical use of the network bandwidth when they are required to convey the same information to several sites. The availability of high speed links with broadcast capacities opens up the possibility of new range of point-to-multipoint applications, such as multimedia conference systems, broadcasting of information, document distribution and communication for distributed systems. However, in high speed environments, as the transmission speed increases the protocol processing overhead at the stations and network nodes becomes a performance bottleneck. To overcome this processing bottleneck, new protocols have been designed with mechanisms specificly conceived for high bandwidth communications. This approach leads to protocols with simple error recovery procedures.

Automatic repeat request (ARQ) schemes are normally employed to guarantee a reliable end-to-end transfer when the network residual error rate does not satisfy the quality of service required by the application. They are simple to implement, robust, reliable and are normally classified in three basic types: stop-and-wait (SW), go-back-N (GB(N)) and selective-repeat (SR).

Variations to the three basic protocols have also been extensively studied. One interesting idea of these variations is to send multiple copies of a message instead of just sending a single copy. This strategy is simple and presents good performance for satellite channels under noisy condition. Sastry's GB(N) [1] has introduced the constinuously repeated retransmission of messages negatively acknowledged, until receiving the positive acknowledgment. Its performance is poor for high speed satellite noisy environment. Morris [2] has improved the Sastry's scheme using the same retransmission procedure, but a buffer is provided at the receiver to store the good messages following an erroneously received one. The SETRAN scheme [3] also stores good messages at the receiver following an error, but it has a different retransmission strategy. The scheme proposed in [4] combines buffer at the GB(N) receiver to store good messages following an erroneously received one with a few copy retransmissions, instead of repeating continuously the same message.

This paper presents and evaluates two point-to-multipoint GB(N) protocols with memory at receivers that uses continuously repeated retransmissions, being suitable for high speed satellite communications. The first one is an extension of the single destination Morris's GB(N). The second one results from slight modifications in the retransmission procedure. After accomplishing a continuously repeated retransmission, the sender retransmits $m$ ($m \geq 1$) copies of messages sent in advance and received in error.

First, environment is explained. Next, the proposed schemes are described and their throughput efficiencies are evaluated using an analytical model based on discrete-time semi-Markov process. Numerical results of the proposed schemes are provided and compared to other multidestination ARQ schemes. Finally, the conclusions are presented.

## ENVIRONMENT

Throughout this paper it is assumed an environment consisting in one transmitter and $M$ receivers. All the data

is transmitted as fixed length ($L$ total bits and $K$ information ones) *information messages* whose transmission time is defined as the time unit.

The probability of an information message arriving with at least one bit error at a given receiver is denoted by $P_{be}$ ($P_{bc} = 1 - P_{be}$), which is independent from message to message and from receiver to receiver. Moreover, the round trip delay is equal to S for all receivers, Furthermore, it is assumed that transmitter and receivers require a $S+1$ message buffer and the return channel is noiseless.

For each protocol, the throughput efficiency will be obtained by assigning states to the set of receiver according to the recovery process conditions and modeling the set of receivers behavior as a discrete-time semi-Markov process, whose state sequence is $X(0), X(1), X(2), X(3), \ldots, X(n), X(n+1), \ldots$ where $X(n), n = 0, 1, 2, \ldots$, is a random variable denoting the state after the reception of the $n^{th}$ message.

## THE PROPOSED PROTOCOLS

### Repeated Copy Scheme

The first protocol, hereafter called repeated copy scheme, is a straightforward point-to-multipoint version of the single destination Morris's GB(N) scheme [2]. Buffering memories are provided at the receivers to store errorless messages sent in advance that are received following erroneous ones. Each receiver begins an error recovery procedure when it detects an erroneous message. It sends a retransmission request, reserves a slot in its buffer and continues to receive, process and acknowledge the messages sent in advance. The messages correctly received are stored to posterior delivery while a buffer slot is reserved for each erroneous detected message. Henceforth, the missing messages corresponding to the buffer slots will be called messages $B_{e_i}$ and their respective retransmission requests, REJ $B_{e_i}$, where $0 \leq i \leq S$. The numeration of messages $B_{e_i}$ takes into account the set of receivers. Thus, after the retransmission request sending, the first message among those sent in advance detected in error will be called message $B_{e_1}$, no matter how many receivers have received it in error. The second will be called message $B_{e_2}$, and so on.

On the receipt of the REJ $B_{e_0}$, the transmitter repeats constinuously the message $B_{e_0}$ until receiving its positive acknowledgment. Then, it starts retransmitting continuously the message $B_{e_1}$, if it exists. Thus, the transmitter accomplishes a continuously repeated retransmission for each message $B_{e_i}$. After recovering all these messages the transmitter starts transmitting new ones.

This protocol defines two states for the set of receivers: normal and exceptional states. In the normal state, the receivers accept the sequenced correctly received messages and deliver them to their users. There is no messages stored in any receiver buffer and no outstanding error recovery in this state. The set of receiver turns to the exceptional state when one or more receivers detect an erroneous message (message $B_{e_0}$). It remains in this state while there is an erroneous message to be recovered. It is defined that the transition occurs at the beginning of this message reception. The set of receivers only returns to the normal state after recovering all messages $B_{e_i}$ using continuously repeated retransmission for each one.

The throughput efficiency of the multipoint repeated copy scheme will be derived. The total protocol efficiency is the weighted mean value of the efficiency in each state:

$$\eta = \frac{K}{L}\left(P(N) + P(E)\frac{A+1}{E[t_E]}\right) \tag{1}$$

where $P(N)$ and $P(E)$ are the probabilities of the set of receivers being in normal and exceptional states, respectively. The expression $(K/L)(A+1)/E[t_E]$ is the exceptional state eficiency, where $A$ is the expected number of messages sent in advance and delivered (sequenced error-free messages) to the user during exceptional state. As the set of receivers passes from normal to exceptional state at the beginning of an erroneous message reception, only error-free messages are received in normal state and its efficiency is $K/L$. Furthermore, the set of receivers passes from exceptional to normal state after recovering all messages $B_{e_i}$ and delivering to their users all $S$ messages sent in advance, consequently $A = S$.

Since the set of receivers always switches back and forth between normal and exceptional states (there are no virtual transitions), the conditional transition probabilities are:

$$P(E \mid N) = P(N \mid E) = 1 \tag{2}$$

Using equations 1 and 2, and applying the semi-Markov process properties, the throughput efficiency can be given by:

$$\eta = \frac{K}{L}\frac{E[t_N] + S + 1}{E[t_N] + E[t_E]} \tag{3}$$

where $E[t_N]$ and $E[t_E]$ are the mean waiting time at the normal and exceptional states, respectively.
The mean waiting time in normal state is:

$$E[t_N] = \sum_{i=0}^{\infty} i.(1 - P_{bc}^M).(P_{bc}^M)^i = \frac{P_{bc}^M}{1 - P_{bc}^M} . \tag{4}$$

Substituting this mean waiting time (equation 4) in equation 3, the throughput efficiency expression becomes:

$$\eta = \frac{K}{L}\frac{P_{bc}^M + (1 - P_{bc}^M).(S+1)}{P_{bc}^M + (1 - P_{bc}^M).E[t_E]} . \tag{5}$$

The mean waiting time at exceptional state is given by:

$$E[t_E] = 1 + S + E[n_e](1 + E[n_{B_{e_i}}]), \qquad (6)$$

where $E[n_e]$ is the expected number of retransmission attempts needed to recover a message $B_{e_i}$ and $E[n_{B_{e_i}}]$ is the expected number of message $B_{e_i}$ with $i \geq 1$, given by:

$$E[n_e] = \sum_{j=1}^{\infty} (S+j) \frac{(1-P_{be}^{j+1})^M - (1-P_{be}^{j})^M}{1-P_{bc}^M}. \qquad (7)$$

and

$$E[n_{B_{e_i}}] = \sum_{i=1}^{S} i \begin{pmatrix} S \\ i \end{pmatrix} (1-P_{bc}^M)^i \, (P_{bc}^M)^{S-i} = S(1-P_{bc}^M). \qquad (8)$$

**Repeated copy plus m-copy scheme**

The second protocol presents a slightly different retransmission procedure when compared to the preceding scheme. After the constinuously repeated retransmission of message $B_{e_0}$, the sender retransmits $m$ ($m \geq 1$) copies of each message $B_{e_i}, i \geq 1$. After retransmitting $m$ copies of all messages $B_{e_i}$, the transmitter sends new messages, not previously transmitted. If these $m$ copies do not successfully recover a message $B_{e_i}, i \geq 1$, this one becomes the new message $B_{e_0}$ and the retransmission recovery is repeated.

This scheme defines three states for the set of receivers: normal, exceptional and transitory ones. In this case, during the exceptional state only the message $B_{e_0}$ is continuously repeated. After accomplishing this continuously repeated retransmission, the transmitter have already received, from all receivers, acknowledgments for all messages sent in advance and correctly received. So, it does not need retransmit these messages. It retransmits only the messages received in error by at least on receiver. Instead of just a single copy, the sender retransmits $m$ ($m \geq 1$) copies of these messages. After retransmitting the copies, it sends new messages. Fig. 1 shows the protocol state diagram.

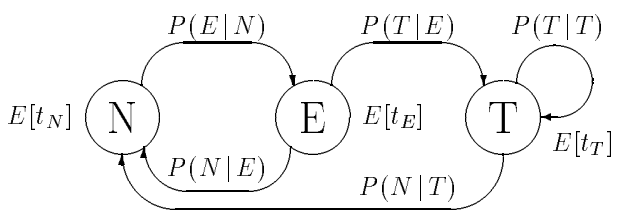


Figure 1: RC + $m$-copy semi-Markov Process model

The set of receivers passes from the exceptional to the normal state if all messages sent in advance are correctly

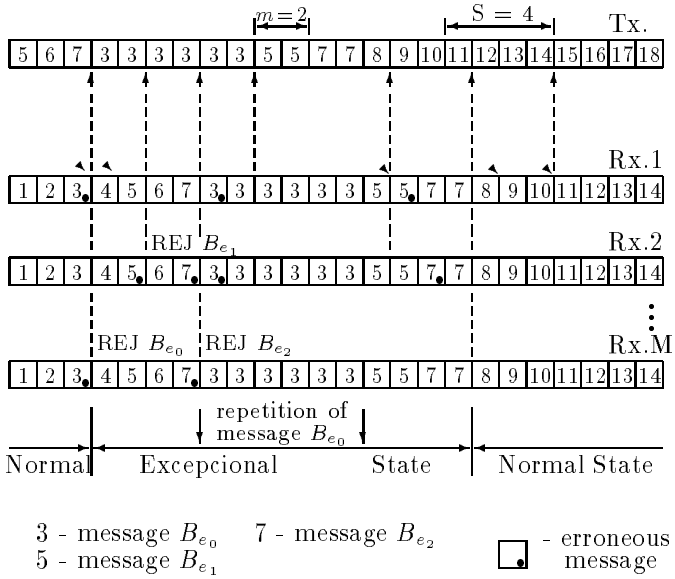


Figure 2: Repeated copy plus m-copy Scheme

by all receivers or if all messages $B_{e_i}$ are recovered by the multiple ($m$) retransmissions. Otherwise, the first not recovered message $B_{e_i}$ becomes the message $B_{e_0}$ and the set of receivers passes to the transitory state. This state is similar to the exceptional one. The diference is that in the transitory state, the message $B_{e_0}$ have already been retransmitted $m$ times. The new message $B_{e_0}$ will be continuously repeated until the transmitter receiving positive acknowledgment from all receivers. Fig. 2 shows the protocol behavior.

The throughput efficiency is given by:

$$\eta = P(N)\frac{K}{L}P_{bc}^M + P(E)\frac{K}{L}\frac{A_E+1}{E[t_E]} + P(T)\frac{K}{L}\frac{A_T+1}{E[t_T]}. \qquad (9)$$

where $A_E$ and $A_T$ are mean number of messages delivered to the user in the exceptional and transitory states, respectively.

## NUMERICAL RESULTS

This section discusses numerical results of the proposed schemes and compares them with other published results. Fig. 3 shows the throughput efficiency, for $M = 10$ and $R = 10Mbps$, of the proposed schemes, full and limited memory GB(N) proposed by Gopal&Jaffe [5] and Broadcast GB(N) with blocks of 1024 ($F = 1024$) messages proposed by Towsley [6]. The RC and limited memory GB(N) protocols present pratically the same results. The RC+m-copy protocol outperfoms all the other protocols and is better than the B-GB(N) for a large region of bit error rate, eventhough this one uses twice more memory at the transmitter. It can be observed that, each of RC+m-copy curve provides an envelope of optimum per-

formance that specifies the values of $m$ to use for a given bit error rate. It can also be observed an efficiency decrease for greater $m$ values in the region of high efficiency value. This can be explained considering that a given $m$ value may be too much for a low BER region and not enough for a high BER region. The improvement due to greater $m$ values occurs in a region of higher BER, where the throughput efficiency can be low and of no practical interest. The results suggest the use of low $m$ values.

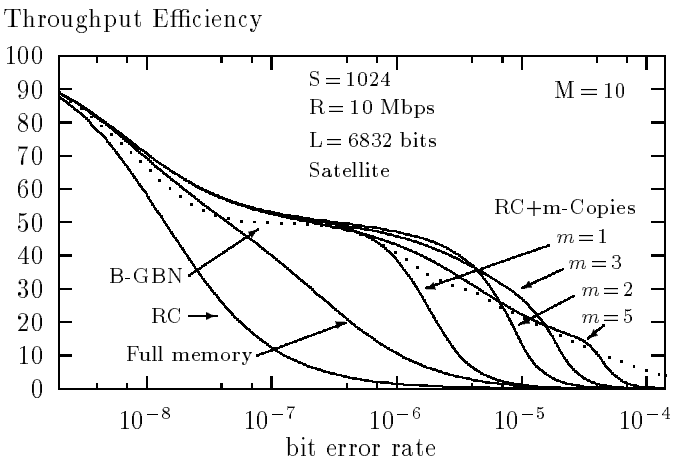


Figure 3: Throughput Efficiency comparing the proposed schemes to other Point-to-Multipoint GB(N) schemes.

Fig. 4 compares the throughput efficiency versus $M$ (number of receivers). It can be observed that RC+m-copy scheme outperforms all the other GB(N) protocols and is the least sensitive to the augmentation of the number of receivers. Furthermore, for this bit error rate ($10^{-6}$) the RC+m-copy scheme performs better than Towsley&Mithal's selective repeat protocol [7] for $M > 27$.

## CONCLUSION

This paper has presented and analysed two point-to-multipoint GB(N) protocols. They employ continuously repeated retransmissions, provide buffers at receivers, are easy to implement and fit well for high speed satellite environments. The proposed strategies do not require too much memory, avoid buffer overflows and performs well even under a noisy condition where the channel presents high error rate. The recovered procedures are simple and the first one can use the accumulative acknowledgment technique that is important for communication with a large number of receivers. The analytical results suggest low values for $m$ in the "repeated plus m-copy" scheme and show that this strategy performs considerably better than many other point-to-multipoint GB(N) results.

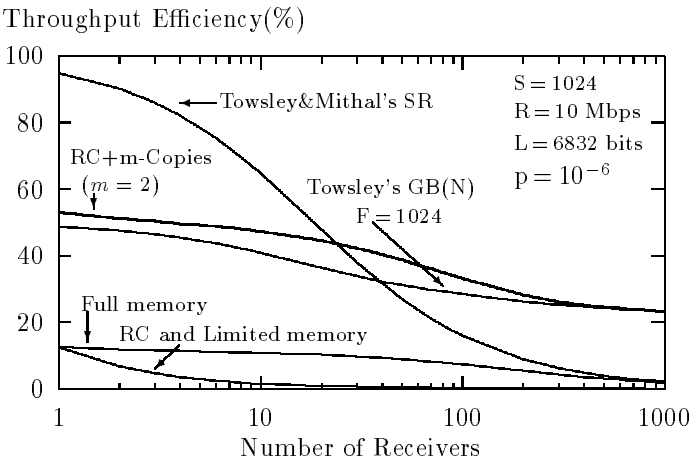


Figure 4: Efficiency *versus* number of receivers.

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