An Accumulative Acknowledgment Protocol for Point-to-Multipoint High Speed Communications

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Abstract

This paper proposes and analyses the performance of a finite buffer point-to-multipoint selective repeat (SR) protocol with accumulative acknowledgment. The proposed strategy is based on three key ideas. The first is the use of accumulative acknowledgments in order to reduce the required processing overhead and implementation complexity. The second and third aim at improving the throughput efficiency: to make the most of the selective repeat recovery procedure and to process and store all messages correctly received during an exception condition. The proposed strategy performs very well under a wide range of conditions, being more efficient than all known point-to-multipoint GB(N) strategies, for low bit error rates.

1 Introduction

Although new technologies have permitted to reduce considerably the transmission errors, they continue to exist in communication systems due to message loss, congestion/flow control, statistics multiplexing and resources (CPU and memory) starvation. Therefore, end-to-end error control is still required for a great number of applications. Automatic-repeat-request (ARQ) is the most common method for handling transmission errors in computer networks since it offers a simpler implementation, performs well on many types of real channels and adapts itself in a satisfactory way to different behaviors on the transmission channel.

Accumulative acknowledgment simplifies the implementation and eliminates the need for an acknowledgment for each received message, allowing the use of different frequency acknowledgment steps, reducing the number of acknowledgment messages and the processing overhead. In point-to-multipoint environments, the accumulative acknowledgment allows the transmitter maintains only a list (or a control variable) for each receiver instead of a list/control variable for each receiver-message pair. This acknowledgment procedure simplification provides an additional reduction on the transmitter processing overhead.

This paper presents and evaluates an accumulative acknowledgment selective repeat scheme suited for point-to-multipoint high speed communications, mainly in environments in which most part of the time the channel bit error rate is very low and occasionally high. It is based on the point-to-point Extended SREJ+REJ protocol [1]. Its key idea is to make the most of the SREJ recovery, to process and to store all non-sequenced messages correctly received during an SREJ/REJ exception condition, for posterior delivery to the user. The proposed strategy performs very well under a wide range of transmission speeds, bit error rates, round trip delays and number of receivers.

Section 2 describes the proposed scheme. Section 3 presents the analytical model based on discrete-time semi-Markov process, and derives a *lower bound* on the throughput efficiency of the new protocol. Numerical results are provided and compared to other ARQ schemes. Finally, conclusions are presented.

2 The Proposed Scheme

It is assumed an environment consisting in one transmitter and M receivers, where all data is transmitted in form of fixed length (L total bits and K information ones) *information messages*, over a point-to-multipoint channel. The L bits transmission time is defined as the time unit.

If a message numbered N(R) is detected in error, the receiver sends, according to certain conditions, either a SREJ N(R) message or REJ N(R) one. The return channel is assumed to be error free.

Furthermore, the probability of an information message arriving with at least one bit error at a given receiver is denoted by P_{be} ($P_{bc} = 1 - P_{be}$). This probability is independent from message to message and from receiver to receiver. The round trip delay, defined as the time from the transmission of a message until the receipt of its acknowledgment, measured in time units, is constant and equals to S for all receivers. Moreover, it is also assumed that: there is always an information message waiting to be transmitted at the transmitter; each receiver has a $S+1$ message buffer and the messages are identified at the instance of arrival, even the erroneous ones.

The proposed strategy is based on the point-to-point Extended SREJ+REJ protocol [1]. The scheme makes the most of the SREJ recovery procedure. The REJ one is only used under certain combination of errors. When a receiver detects an error while no outstanding SREJ/REJ exception condition exists, then it sends SREJ to recover. Otherwise, when a receiver detects an error while an outstanding SREJ recovery procedure exists, then the REJ message is sent. Due to the accumulative acknowledgment, the receiver only sends this REJ after receiving the retransmission of the requested message. The REJ is also sent in case of error in the requested message retransmission.

In order to ensure that every message is correctly received by all receivers, the transmitter maintains a state variable $V_i(R)$ for each receiver, indicating the number of the last message positively acknowledged by this receiver. Hence, $V_i(R) = N(R)$ indicates the receiver i has correctly received all messages with sequence number smaller than N(R). Besides these variables, the transmitter maintains two global state variables: $VG(S)$ indicates the number of next message to be transmitted and $VG(R) = \min_i\{V_i(R)\}$ indicates the last message positively acknowledged by all receivers. Thus, $VG(R) = N(R)$ indicates all messages with sequence number smaller than N(R) have been received correctly by all receivers and may be removed from retransmission buffer.

When a receiver sends an SREJ message beginning a selective retransmission error recovery, it reserves a slot in its buffer and continues to receive, process and acknowledge the messages sent in advance. The messages correctly received are stored to posterior delivery while a buffer slot is reserved for each detected erroneous message. An *SREJ exception condition* exists in this receiver from the time SREJ is sent until the time the requested message retransmission is received. Henceforth, the missing messages corresponding to the buffer slots reserved at the receivers will be called messages B_{e_i} and the corresponding retransmission requests, either SREJ B_{e_i} or REJ B_{e_i}, where $0 \leq i \leq S$. The numeration of messages B_{e_i} takes into account the set of receivers. Therefore, after the first SREJ sending corresponding to SREJ B_{e_0}, the first message, out of those sent in advance, detected in error by one or more receivers will be called message B_{e_1}, regardless which and how many receivers have received it in error. The second will be called message B_{e_2}, and so on. In each receiver only one SREJ recovery in progress can exists whereas in the whole set of receivers it is possible to have many simultaneous outstanding SREJ exception conditions, because many messages can be received in error by the set of receivers while each individual receiver detects no more than one erroneous message.

A receiver sends a REJ requesting a GB(N) retransmission in two cases: (a) the retransmission requested by the SREJ is unsuccessful and (b) the retransmission requested by the SREJ is successful, but at least one message among those sent in advance is detected in error, during the SREJ exception condition. In the second case, after sending a REJ the receiver can receive and store new messages correctly received among those sent in advance as well as can reserve additional buffer slots while waiting for the requested retransmission. It is defined that a *REJ exception condition* exists in this receiver from the moment it sends the REJ until the requested message and all erroneous messages, among those sent in advance, are recovered. When message B_{e_i} retransmission is received in error, another REJ B_{e_i} is sent. In this case while waiting the additional retransmission of this message, the receivers process the retransmission of messages sent in advance (messages numbered from $B_{e_i}+1$ until $B_{e_i}+S$) which can recover messages $B_{e_{i+1}}, B_{e_{i+2}}, \ldots$. If retransmission of the message B_{e_i} is successful and the retransmission of the one $B_{e_k}, k > i$ is unsuccessful, a REJ B_{e_k} is sent requesting a go-back-N retransmission beginning with the message B_{e_k}. This REJ B_{e_k} is sent by the receivers that had not recovered this message B_{e_k}.

On the transmitter side, the retransmission procedure is chosen based on the acknowledgments of all receivers and on the absence of REJ recovery in progress. If there is no REJ recovery in progress, the transmitter retransmits a copy of the message B_{e_i}, if it receives an SREJ B_{e_i} of at least one receiver and no REJ B_{e_i}. Otherwise, if it receives a REJ B_{e_i} of at least one receiver, it accomplishes a REJ retransmission beginning with the message B_{e_i}. After receiving a REJ B_{e_i} and executing a GB(N) retransmission, the transmitter will disregard other retransmission requests received before retransmitting all messages stored in its retransmission buffer (messages sent in advance with respect to the message B_{e_i}). Nevertheless, the number B_{e_i} contained in the received control messages (REJ, SREJ and RR) will be used to updated the variables $V_j(R)$.

3 Performance Analysis

The protocol throughput efficiency is defined as the ratio between the number of information bits communicated to the set of receivers and the total number of transmitted and retransmitted bits (including overhead bits). It will be obtained by assigning states to the set of receivers according to the recovery process conditions in at least one receiver and modeling the set of receivers behavior as a discrete-time semi-Markov process, whose state $X(n), n = 0, 1, 2, \ldots$, denotes the state of the set of receivers after the reception of the n^{th} message.

The proposed strategy defines three states to the set of receivers: normal, exceptional and transition states. In normal state there is no exception condition in the set of receivers (there is no outstanding error recovery in any receiver) and there are no messages stored in any receiver buffer. The receivers accept the sequenced correctly received messages and deliver them to the users.

The set of receivers is said to be in exceptional state when an SREJ exception condition exists in at least one receiver and no REJ exception condition exists in any receiver. There is at least one receiver waiting for recovery of a message with sequence number between B_{e_0} and $B_{e_0}+S$. The set of receivers passes from normal to exceptional state when one or more receivers detect an erroneous message (message B_{e_0}) and send a SREJ B_{e_0} message requesting a selective retransmission. It is defined that the transition occurs at the beginning of this message reception. The set of receivers continues in this state either until recovering all messages B_{e_i} using selective retransmission only or until a receiver sending a REJ message.

The set of receivers is said to be in transition state when a REJ exception condition exists in at least one receiver. That is, if there is a REJ recovery in progress. The set of receivers goes back from exceptional to normal state if all the detected erroneous messages are recovered by SREJ retransmissions only. That is, if all SREJ recoveries are successful (all erroneous messages are recovered) without any receiver detecting a multiple error condition (more than one erroneous message). Hence, a transition from the exceptional to the transition state occurs if either an SREJ B_{e_i} recovery is unsuccessful or at least one receiver detects more than one erroneous messages during the SREJ recovery (including the message B_{e_0}).

When a receiver sends a REJ B_{e_i} message, the set of receivers goes to the transition state which will be classified in cycles: one *transmission cycle* and one or more *retransmission cycles*. The transmission cycle corresponds to the time interval between the REJ B_{e_i} issue and the first retransmission of messages B_{e_i}. If the SREJ B_{e_0} recovery had been unsuccessful, during the transmission cycle the set of receivers receives either messages which will be discarded due to buffer overflow or retransmission of messages B_{e_j} requested by previous SREJ B_{e_j} issues. Otherwise, if the SREJ B_{e_0} had been successful, the set of receivers can store new messages correctly received during this cycle, since the recovery of B_{e_0} delivers spaces in receiver buffers. In this case, new B_{e_i} messages can be identified. The number of retransmission cycles corresponds to the number of GB(N) retransmissions needed to recover either erroneous messages not recovered by SREJ retransmissions or new messages B_{e_i} identified during the transmission cycle. Hence, the set of receivers only returns to the normal state after recovering all these messages B_{e_i}. Each retransmission cycle ends after the reception of the the message $B_{e_k}+S$ retransmission, where B_{e_k} is the number of the first message requested by the REJ sent in the beginning of the transition state. During the transition state, the set of receivers stores, recovers and/or delivers only the messages corresponding to the transmission cycle (messages with sequence number between B_{e_k} and $B_{e_k}+S$). Thus, during this state $S+1$ messages are always communicated to set of receivers.

In the following paragraphs, the throughput efficiency of the this scheme will be derived. As the set of receivers passes from the normal to the exceptional state at the beginning of the erroneous message reception, only error-free messages are received in the normal state. Then, the throughput efficiency of the protocol in this state is K/L.

As $S+1$ messages are always communicated during the transition state, the total protocol efficiency, which is the weighted mean value of the efficiency in each state, is given by:

$$\eta = \frac{K}{L}\left(P(N) + P(E)\frac{A}{E[t_E]} + P(T)\frac{S+1}{E[t_T]}\right) \qquad (1)$$

where $P(N), P(E)$ and $P(T)$ are the probabilities of the set of receivers being in normal, exceptional and transition states, respectively. The expressions $(K/L)(A/E[t_E])$ and $(K/L)((S+1)/E[t_T])$ are the efficiencies for the exceptional and transition states, respectively. A is the average number of messages "sent in advance" and delivered to the user during the exceptional state. $E[t_E]$ and $E[t_T]$ are the mean waiting time at the exceptional and transition states, respectively.

The mean waiting time in normal state is:

$$E[t_N] = \sum_{i=0}^{\infty} i.(1-P_{bc}^M).(P_{bc}^M)^i = \frac{P_{bc}^M}{1-P_{bc}^M} \qquad (2)$$

Using equation 2 and applying the semi-Markov process properties, the throughput efficiency, as a function of $P(T\,|\,E), A, E[t_E]$ and $E[t_N]$, can be expressed as:

$$\eta = \frac{K}{L}\,\frac{P_{bc}^M + (1-P_{bc}^M)(A + P(T\,|\,E)P(T\,|\,E)(S+1))}{P_{bc}^M + (1-P_{bc}^M).(E[t_E] + E[t_T].P(T\,|\,E))}. \qquad (3)$$

where $E[t_E]$ and $E[t_T]$ are the mean waiting time at exceptional and transition states, respectively and $P(T \mid E)$ is the conditional transition probability from exceptional to transition state ($P(T\,|\,E) = 1 - P(N\,|\,E)$).

Next, it will be derived the expressions for $P(T\,|\,E), A, E[t_E]$ and $E[t_N]$. First, the conditional transition probability from exceptional to normal state is derived, which is divided in cases. One of the main cases is the one in which the SREJ B_{e_0} recovery is successful and all messages sent in advance, with respect to B_{e_0}, are correctly received by all M receivers. This case occurs with probability:

$$p_0 = P_{bc}^{M.S} q \qquad (4)$$

where q is given by

$$q = \frac{(1-P_{be}^2)^M - P_{bc}^M}{1-P_{bc}^M}. \qquad (5)$$

The others are the cases in which one or more messages sent in advance are received in error by the whole set of receivers ($B_{e_1}, \ldots$), each individual receiver detects at most one erroneous message (including (B_{e_0}) and all messages $B_{e_i}(i \geq 0)$ are recovered by SREJ retransmission only. In other words, in these cases each receiver that detects the message B_{e_i} in error, ($i \geq 0$), receives correctly all other messages $B_{e_j}(j \neq i)$ that had been received in error by other(s) receiver(s).

This analysis takes into account only the cases where the waiting time in the exceptional state is not greater than $2(S+1)$. In these cases the set of receivers both detects $k+1$ messages B_{e_i} (including the message B_{e_0}) during the exceptional state whose SREJ retransmissions are successful and receives correctly all messages with sequence number $N(R)$ so that $B_{e_0}+S \;<\; N(R) \;\leq\; B_{e_k}+S-k$. The probability of occurring the case with $k+1$ messages B_{ei} is:

$$p_k = (1-P_{bc}^M)^k P_{bc}^{M(S-k)} q^{k+1} p_y(k) \sum_{i=k}^{S} \binom{i-1}{k-1} (P_{bc}^M)^{i-k}, \qquad (6)$$

where $p_y(k)$ is the probability of two messages being not received in error by a same receiver, given that B_{e_0} and B_{e_k} are detected by the set of receivers. $p_y(k)$ is given by:

$$p_y(k) = \left(\sum_{j_1=1}^{M} \binom{M}{j_1} P_{be}^{j_1} P_{bc}^{M-j_1} \cdots \sum_{j_{k+1}=1}^{M-nj_k} \binom{M-nj_k}{j_{k+1}} P_{be}^{j_{k+1}} P_{bc}^{M-j_{k+1}} \right) \left(1-P_{bc}^{M}\right)^{-(k+1)} \quad (7)$$

where

$$nj_k = j_1 + \ldots + j_k. \quad (8)$$

The conditional transition probability from exceptional to transition state is

$$P(T\,|\,E) = 1 - P(N\,|\,E) = 1 - \sum_{k=0}^{S} p_k. \quad (9)$$

Next, the expected number of messages delivered during exceptional state, A, and the mean waiting time in this state are derived. A consists in two terms:

$$A = q.A_1 + q.A_2. \quad (10)$$

The term A_1 represents the expected number of messages, out of S ones sent in advance, with sequence number from B_{e_0} until $B_{e_0} + S$, delivered to the user during the exceptional state, given that the SREJ B_{e_0} retransmission is successful. It is given by:

$$\begin{aligned} A_1 = & \sum_{i=0}^{S} \left(1 + \frac{S-i}{i+1}\right) \binom{S}{i} (1 - P_{bc}^{M})^{i} (P_{bc}^{M})^{S-i} + \\ & qP_y(1) \sum_{i=1}^{S} \frac{S+1}{i+1} \binom{S}{i} (1-P_{bc}^{M})^{i} (P_{bc}^{M})^{S-i} + \ldots + \\ & q^{S-1} P_y(S-1) \sum_{i=S-1}^{S} \frac{S+1}{i+1} \binom{S}{i} (1-P_{bc}^{M})^{i} (P_{bc}^{M})^{S-i} + \\ & q^{S} (1 - P_{bc}^{M})^{S} p_y(S). \end{aligned} \quad (11)$$

The term A_2 represents the expected number of messages with sequence number greater than $B_{e_0} + S$ delivered to the user during the exceptional state, given that the SREJ B_{e_0} retransmission is successful. It is assumed that in case of transition from exceptional to transition state, the set of receivers sends the REJ message when it receives the SREJ B_{e_0} retransmission. A_2 is:

$$A_2 = \sum_{k=1}^{S-1} \left(1-P_{bc}^{M}\right)^{k} \left(P_{bc}^{M}\right)^{S-k} q^{k} p_y(k) \sum_{i=k+1}^{S} \binom{i-1}{k} \left(P_{bc}^{M}\right)^{i-k}. \quad (12)$$

On the other hand, the mean waiting time in exceptional state is

$$E[t_E] = S + 2 + A_2 + E[t_a], \quad (13)$$

where A_2 is the additional waiting time due to the transitions that occur after the reception of message B_{e_0} retransmission and $E[t_a]$ is the extra waiting time corresponding to the time units wasted by the SREJ B_{e_i} retransmissions, $i > 0$. $E[t_a]$ is given by:

$$E[t_a] = \sum_{k=1}^{S} k.p_k. \quad (14)$$

Finally, the mean waiting time in transition state is given by:

$$E[t_T] = (1 + E[n_e]).(S + 1). \quad (15)$$

where the expected number of retransmission cycles is given by:

$$E[n_e] = \sum_{j=1}^{\infty} j Pr\{n_i = j\} \quad (16)$$

where the probability that all $S+1$ messages, corresponding to the transmission cycle, are successfully communicated to all M receivers during the j^{th} retransmission cycle, but not all recovered within the $(j-1)^{th}$ retransmission cycle is given by:

$$Pr\{n_i = j\} = Pr\{n_i \leq j\} - Pr\{n_i \leq j-1\} \quad (17)$$

and

$$Pr\{n_i \leq j\} = \frac{(1 - P_{be}^{j+1})^{M} - P_{bc}^{M}}{1 - P_{bc}^{M}} (1 - P_{be}^{j+1})^{M.S} \quad (18)$$

Therefore, substituting equations 15, 13, 10 and 9 in 3, the throughput efficiency of the proposed scheme is obtained.

Throughput Efficiency

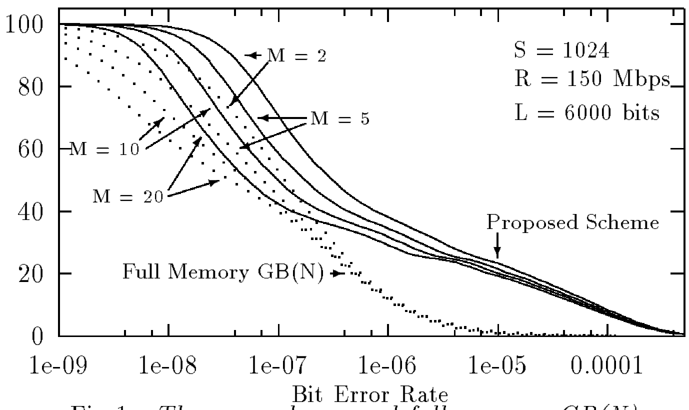

Fig.1 - *The new scheme and full memory GB(N) mboxthroughput efficiency*

4 Numerical Results

This section discusses numerical results of the proposed scheme and compares them with other published results. Throughput efficiency of the proposed scheme and of the *full memory* GB(N) of Gopal&Jaffe[2] is shown in Fig. 1

for various numbers of receivers, $M = 2, 5, 10$ and 20, over a terrestrial channel with at $150\,Mbps$. It can be observed that, for the same number of receivers, the proposed scheme presents a better performance, mainly in low error rate ranges. The SREJ retransmission increases the throughput efficiency at low error rates and receiver buffering improves it at high bit error rates (BER).

Fig. 2 compares, for point-to-multipoint communications at 150 Mbps, the throughput efficiency of the proposed scheme and following protocols: Chandran&Lin's SR [3]; Towsley&Mithal'SR [4] with $q = 1$ and one copy retransmission ($n_1 = 1$); Towsley's GB(N) [5] with blocks of 1024 messages ($F = S = 1024$) and two Gopal&Jaffe's GB(N) [2] (limited memory and full memory GB(N)). It can be noted that in low error rate range, the proposed scheme outperforms the Chandran&Lin'SR and all GB(N) schemes. It can also be observed that for low error rates, the proposed scheme performs equal to the Towsley&Mithal's SR one and outperforms it for a wide range of high error rates.

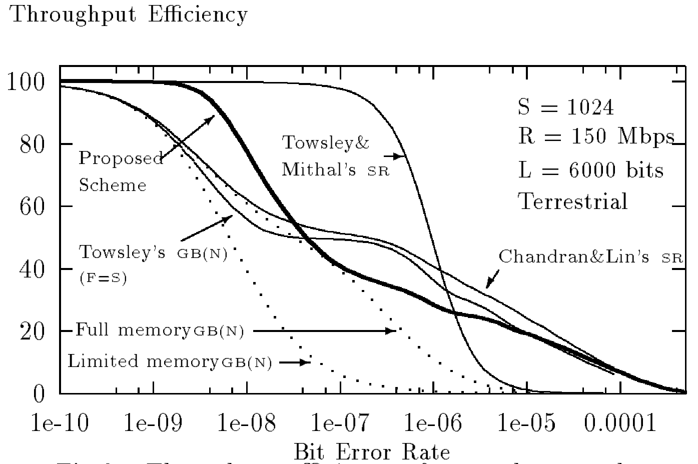

Fig.2 - *Throughput efficiency of new scheme and some GB(N) and SR protocols*

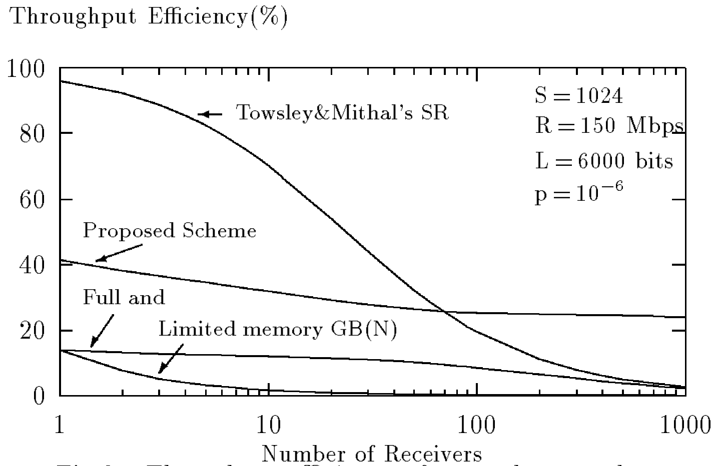

Fig.3 - *Throughput efficiency of new scheme and some GB(N) and SR protocols*

Fig. 3 plots, for point-to-multipoint communications at 150 Mbps, the throughput efficiency of various protocols, as a function of the number of receivers: the proposed scheme, Towsley&Mithal's SR, full memory and limited memory GB(N). It can be observed that proposed scheme, is the least sensitive to the augmentation of the number of receivers and performs better than both the full memory GB(N) (for any number of receivers) and Towsley&Mithal's SR for $M > 70$.

5 Conclusion

This paper has presented and analysed a selective repeat scheme with accumulative acknowledgments for high speed point-to-multipoint communications. It combines the use of both SREJ and REJ recovery procedures and makes the most of the receiver buffering. The use of accumulative feature allows the reduction of the number of acknowledgment messages and control variables in order to minimize both the processing overhead and the implementation complexity. Furthermore, the extended non-sequenced message storage improves the performance, since it reduces the number of errorless discarded messages.

The analytical results have shown that the proposed strategy is considerably better than the Gopal&Jaffe's full memory GB(N) and outperforms the Towsley&Mithal's finite buffer point-to-multipoint SR ARQ scheme for a wide range of high bit error rates and/or large number of receivers. At the same time, for ranges of both low and high bit error rates, its performance is close to the best achievable one by an ARQ protocol.

The proposed protocol seems to be quite robust and numerical results suggest its utilization in end-to-end high speed communications where the error rate is low most of the time and noisy during very short intervals.

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